A High-efficiency and High-accuracy Distance Measurement Technique Based on Phase Differentiation and Accumulation with FMCW radars

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***Abstract*—In the Frequency-Modulated Continuous-Wave (FMCW) radar, high-accurate distance measurement based on frequency estimation suffers from limited frequency resolution and large compute resource consumption. In this paper, a Phase Differentiation and Accumulation (PDA) technique is proposed for high-efficient and frequency resolution free distance measurement with FMCW radar. It works by accumulating center phase variation from a uniformly segmented chirp. The coarse frequency estimated from the segmented chirp signal is used to resolve the phase ambiguity. Theory analysis shows that by properly choosing the segmentation number as 3, the distance estimation variance with the proposed technique can be close to the Cramer-Rao Lower Bound (CRLB) of the conventional frequency estimation method, and the time complexity of the proposed technique is only *O(Nlog(N/3))*, which is even smaller than the non-zero-padded FFT's time complexity. Simulations and experiments show that when achieving similar accuracy, the proposed technique saves over 500 times compute resources.**

***Keywords*—distance measurement, efficiency, FMCW radar, frequency resolution, phase.**

I. Introduction

Nowadays, FMCW radar is widely entering our daily life for different applications such as automotive applications, gesture sensing, in-door activity monitoring, vital sign monitoring, and so on [1-7]. Among these applications, target distance is basic information that the FMCW radar provides.

The conventional distance measurement method is based on the FFT-based frequency estimation [6-7]. However, this method suffers from limited frequency resolution and thus limited distance estimation accuracy. To improve the frequency resolution, a large number of zero is usually padded to the IF signal before FFT, which consumes large compute resources and power, making it inappropriate for the device with limited compute resources such as the embedded device widely used for radar signal processing. In [8-9], different interpolation methods are used to mitigate the compute resource consumption. These methods work by directly interpolating points to a low-frequency-resolution spectrum whose generation requires less compute resources. In [10], a chirp z-transform (CZT)-based technique is also used to reduce the compute resource consumption. It works by performing further CZT in the concerned narrow frequency band to realize a better frequency resolution. However, these interpolation-based or CZT-based methods introduce new errors and still generate limited frequency resolution.

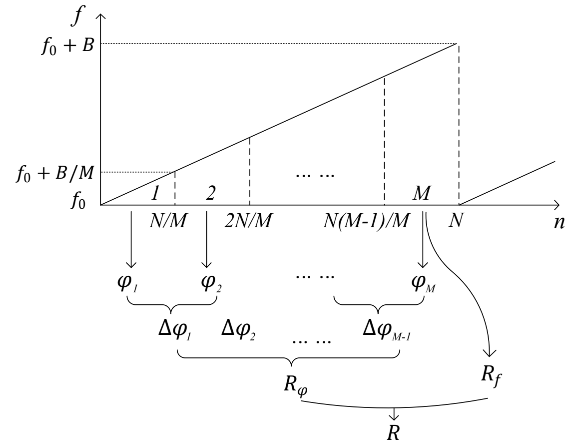

Fig. 1. A chirp signal segmented to be M segments and the simplified procedures of the proposed technique. f_0 is the start frequency and B is the bandwidth.

Moreover, their compute resource consumption is still much larger than the non-zero-padded FFT.

In this work, a high-efficient and frequency-resolution free PDA technique is proposed for accurate distance measurement with FMCW radar. A coarse distance estimated with frequency and a fine distance estimated with the accumulated center phase variation is extracted from the uniformly segmented chirp signal. The two distance is then combined to realize a high-accurate distance measurement. The phase estimation breaks the frequency resolution and the segmented frequency estimation largely reduced the compute resource consumption. Simulations and experiments are carried out to validate the proposed technique.

II. Theory and Analysis

A. Theory

In a common FMCW radar, the IF signal, also known as the beat signal, can be modeled as follows:

$$s_b(t) = A\,exp\left(j\left(\frac{4\pi BR}{Tc}\cdot t + \frac{4\pi f_c R}{c}\right)\right), t \in [\frac{T}{2}, \frac{T}{2}], \qquad (1)$$

where A is the signal amplitude, B is the sweep bandwidth, T is the pulse repetition time (PRT) and, c is the speed of the light, f_c is the central frequency, and R is the target distance.

The frequency component also known as the beat frequency is $f_b = 2BR/Tc$ and the phase item is $\varphi = 4\pi f_c R/c$. The target distance R can be derived from the beat frequency, which is usually estimated with zero-padded FFT. However, as discussed in *Section I*, the frequency resolution of the spectra generated with FFT is limited and the zero-padded FFT consumes large compute resources. Interpolation methods or the CZT [8-10] might help but still has limitation. In this work, a high-efficient and frequency-resolution free PDA technique is proposed for accurate distance measurement with FMCW radar. The technique procedure is shown in Fig. 1 and is detailed as follows.

1) The sampled beat signal of one chirp has a length of *N*. Divide the chirp into *M* segments. Each segment can be seen as a sub-chirp with bandwidth being $\Delta f = B/M$ and center frequency being $f_{cm} = f_0 + Bm/M$, *m=1~M*. Thus, its distance resolution is $\Delta R = c/2\Delta f$.
2) Perform non-zero-padded FFT on each segment to get the spectra $F_m[k]$, *k=1~N/M*, *m=1~M*. The distance between two adjacent points of $F_m[k]$ is ΔR. Find the target peak's location of k_b. The target distance can be coarsely estimated to be $R_f = \Delta R \cdot k_b$.
3) Extract the phase of $F_m[k_b]$ to get $\varphi_m = arg(F_m[k_b])$, *m=1~M*. The adjacent phase difference is (*m=1~M-1*):

$$\Delta\varphi_m = \begin{cases} \varphi_{m+1} - \varphi_m, \ \varphi_{m+1} > \varphi_m \\ \varphi_{m+1} + 2\pi - \varphi_m, \ \varphi_{m+1} < \varphi_m \end{cases}. \tag{2}$$

4) Accumulate the phase difference to get: $\Delta\varphi = \Delta\varphi_1 + \Delta\varphi_2 + \cdots \Delta\varphi_{M-1} = \varphi_M - \varphi_1 + p \cdot 2\pi$, where p is the number of 2π that is added in $\Delta\varphi_1 \sim \Delta\varphi_{M-1}$. Therefore, the fine distance with ambiguity can be estimated to be: $R_\varphi = c\Delta\varphi/4\pi(f_{cM} - f_{c1})= c\Delta\varphi/4\pi(M-1)\Delta f$. This is similar to the FSK CW radar with the lower frequency being f_{c1}, the upper frequency being f_{cM}, and unambiguous phase range being *0~2π(M-1)*. Therefore, the unambiguous range of R_φ is $0\sim c/2\Delta f$, which is the same as the distance resolution ΔR.
5) The absolute fine distance can then be estimated with the combination of R_f and R_φ:

$$R = \begin{cases} R_f + R_\varphi, \ F_m[k_0+1] \geq F_m[k_0-1] \\ R_f + R_\varphi - \Delta R, \ F_m[k_0+1] < F_m[k_0-1] \end{cases}. \tag{3}$$

Since R_φ is not frequency resolution limited, the distance R estimated with the proposed technique is not frequency resolution limited.

B. Noise analysis and time-complexity analysis

The fine distance is estimated as $R_\varphi = c(\varphi_M - \varphi_1 + p \cdot 2\pi)/4\pi(M-1)\Delta f$, which means the proposed technique's accuracy is directly related to φ_M and φ_1. As indicated in [7], the variance of the phase item φ_m under an *SNR* of η is:

$$Var(\varphi_m) = \frac{1}{\eta N_1}, \tag{4}$$

where $N_1 = N/M$ is the segment length. Therefore, the variance of $\Delta\varphi$ is:

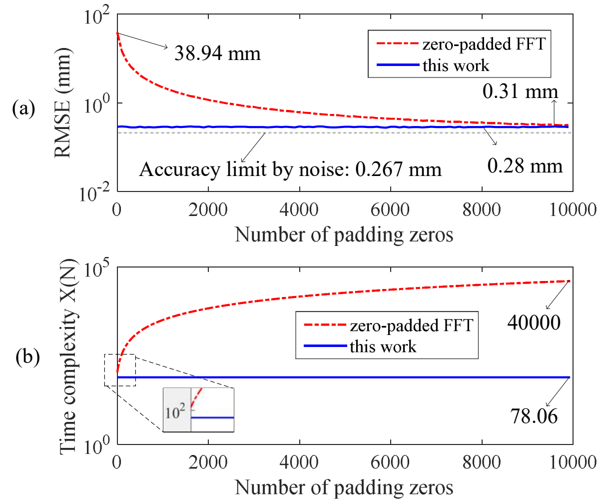

Fig. 2. Simulated distance measurement results with 120 GHz FMCW radar: (a)The distance estimation RMSE and (b) the time complexity with the conventional zero-padded FFT method and the proposed technique.

$$Var(\Delta\varphi) = Var(\varphi_M) + Var(\varphi_1) = \frac{2}{\eta N_1}. \tag{5}$$

Thus, the variance of the R_φ is

$$Var(R_\varphi) = \frac{c^2}{(4\pi)^2 B^2 (1-N_1/N)^2} \cdot \frac{2}{\eta N_1}, \tag{6}$$

It can be seen from (6) that $Var(R_\varphi)$ varies with N_1. With simple analysis, it can be found that the minimum $Var(R_\varphi)$ is obtained when $N_1 = N/3$, which means *M=3*. The minimum $Var(R_\varphi)$ can then be derived as:

$$Var(R_\varphi) = \frac{27c^2}{32\pi^2 \eta N B^2}, \tag{7}$$

which is close to the CRLB of the conventional frequency estimation method [7]:

$$Var(R) = \frac{24c^2}{32\pi^2 \eta N B^2}. \tag{8}$$

Moreover, with *M*=3, only 3 *N*/3-points FFT is needed in the proposed technique, which means the time complexity of the proposed technique is only *O(Nlog(N/3))*. This is even smaller than the original non-zero-padded FFT's time complexity of *O(Nlog(N))*.

III. Simulations

Simulations are carried out in MATLAB to validate the proposed technique. In order to make the simulations consistent with the experiment below, the simulated radar parameters are set to be the same as the experiments. The center frequency is set to 120 GHz. The bandwidth is set to be 4 GHz. The sampling rate is set to be 10 kHz. Additive white Gaussian noise is added to the IF signal with the awgn() function in MATLAB. The *SNR* is set to be 20 dB. Therefore, according to (7) and (8), the accuracy limits of the conventional zero-padded FFT method and the proposed technique are 0.267mm and 0.281mm, respectively.

Fig. 2(a) shows the distance estimation root-mean-square-error (RMSE) with the conventional zero-padded FFT method and the proposed technique. Different padding zeros are applied to the conventional method, which leads to different RMSE and different compute resource consumption. For each data point in Fig. 2(a), the point target is positioned randomly along the range axis for each measurement and the RMSE is estimated with 10000 measurements. It is seen in Fig. 2(a) that when no zero is padded, the RMSE is around 38.93 mm, which is limited by the frequency resolution of 37.5 mm. With the padding zero number increased, frequency resolution limitation decreases and finally comes to the accuracy limits caused by the noise, which is around 0.267 mm as discussed previously. On the other hand, no frequency resolution limit is observed with the proposed technique and the result is close to the noise accuracy limit of 0.28 mm.

Fig. 2(b) shows the time complexity of the conventional zero-padded FFT method and the proposed technique. The time complexity of the conventional zero-padded FFT is estimated as $X(N_0) = N_0 log(N_0)$ with N_0 being the zero-padded signal length and the time complexity of the proposed technique is estimated to be $X(N) = N log(N/3)$ with N being the original chirp length. It is seen in the inset that the time complexity of the proposed technique is even smaller than the conventional method at the curve start when no zero is padded. With more zero-padded, the conventional method's accuracy increases, but its time complexity also increases fast. In order to realize the range accuracy close to the proposed technique, around 9940 zeros are padded to the original signal, which causes a time complexity of 40000. This is around 512.4 times of proposed technique's time complexity of 78.06.

IV. Experiments

Experiments with a custom-built 120 GHz FMCW radar are carried out in the office environment to validate the proposed technique. The experimental setup is shown in Fig. 3. As the inset shows, the radar is made with the radar front-end TRA-120-001 (Silicon Radar) and the PLL chip ADF4159. The *I/Q* signal is sampled by the data acquisition (DAQ) board (National Instruments) for post-signal processing in the computer. The radar sits around $d_o = 56\,cm$ away from the linear stage (Zaber X-LDM060C-AE54D12). The radar bandwidth is set to 4 GHz, the PRT is set to 6 ms, and the sampling rate is set to 10 kHz. The linear stage is programmed to perform a sinusoidal movement of 2cm@0.01Hz. The slow frequency is to avoid the influence of the Doppler effect.

Fig. (4) shows the measured distance curve. The Blackman window function is applied to mitigate the influence of the leakage and clutters. It can be seen in Fig. 4(a) that, when the signal length is padded to 100, the accuracy is largely limited by the frequency resolution and the RMSE is 23.4 mm. By increasing the signal length to 1000, the accuracy is largely improved to an RMSE of 2.43 mm, but the data processing time also increases and the distance value is still discrete in this situation as shown in Fig. 4(b). By further increasing the signal length to 10000 as shown in Fig. 4(c), the discrete phenomenon disappears which means the accuracy comes to

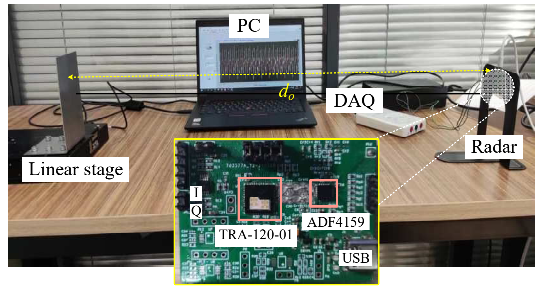

Fig. 3. The experimental setup of distance measurement with a custom-built 120 GHz FMCW radar and a linear-stage Zaber X-LDM060C-AE54D12. The inset shows the detail of the radar.

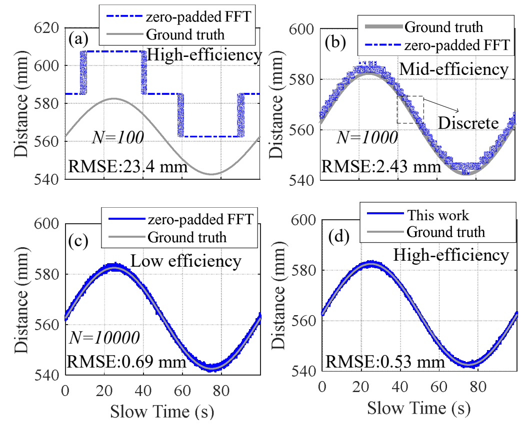

Fig. 4. The measured distance (a)/(b)/(c) with the conventional zero-padded FFT and (c) the proposed technique. In (a)/(b)/(c), the signal is padded to a length of 100/1000/10000.

the limit caused by the noise. The RMSE is decreased to around 0.69 mm. However, to achieve this accuracy, the compute resource consumption is over 100 times the situation in Fig. 4(a). On the other hand, with the proposed technique, as shown in Fig. 4(d), the RMSE of the measured distance can be smaller than the RMSE of Fig. 4(c), which is only around 0.53 mm, and the compute resource consumption can be close to the situation of Fig. 4(a).

V. Conclusion

This paper presents a high-efficient and frequency-resolution free PDA technique for accurate distance measurement with FMCW radar. It achieves similar accuracy of the conventional FFT-based distance estimation method while saving over 500 times of compute resources. The technique is especially suitable for the embedded device which is widely used for radar systems but only has a limited compute resource. It should be noted that the proposed algorithm is based on the center phase variation, which makes its accuracy only comparable to the frequency evaluation method [6]. By combining the center phase as in [6-7], the accuracy of the proposed technique can be further improved.

Acknowledgments

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