

Characterizing Precision Coaxial Air Lines as Reference Standards for Cryogenic S-Parameter Measurements at Milli-kelvin Temperatures

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Abstract—Accurate S-parameter characterization of coaxial microwave devices and interfaces at milli-kelvin (mK) temperatures is essential for building high performance quantum computing systems. Coaxial air-dielectric transmission lines (air lines) are used routinely as primary reference standards for S-parameter measurements at room temperature. For the first time, the feasibility of utilizing such air lines as primary reference standards at mK temperatures is investigated. However, for air lines to be used at such temperatures, they must first be characterized accurately in terms of their characteristic impedance and propagation constant. This paper describes techniques for characterizing air lines at both room temperature (taken here to be 296 K) and mK temperature (around 25 mK). The measurement results demonstrate the feasibility of using air lines as primary reference standards at mK temperatures.

Keywords—Cryogenic measurements, calibration, precision coaxial air lines, scattering parameters, vector network analyzer.

I. INTRODUCTION

Development of large-scale quantum computing systems will require calibrated microwave measurements at milli-kelvin (mK) temperatures, including S-parameter measurements. Such measurements are necessary for accurate characterization of quantum microwave circuits, components and waveforms used for optimization of quantum or classical components interfacing with quantum computing systems.

Recent research has focused on developing S-parameter measurement capabilities at cryogenic temperatures down to tens of mK [1]-[4]. Calibration is an essential step in the measurement process in order to mathematically correct the non-idealities in the measurement setup caused by imperfect components and connector interfaces, lossy cabling and attenuators on the input side, and the response of the vector network analyzer (VNA). The work in [1]-[2] focused on characterizing non-coaxial devices such as RF integrated circuits and superconducting quantum integrated circuits. In [3], an S-parameter measurement system was proposed for characterizing coaxial connectorized components at mK temperatures. A commercial SMA adaptor with a PTFE dielectric filling and a characteristic impedance of approximately 50 Ω at room temperature was used to set the reference impedance of the calibration at mK temperatures. In [4], a one-port cryogenic coaxial calibration scheme employing the Short-Open-Load (SOL) technique was proposed, where the characteristic impedance for the S-parameter measurements was referenced to the impedance of an SMA-based 50 Ω matched load. However, the properties of the dielectric used in these devices may change at mK

temperatures, thus affecting the reference impedance of the calibration, and therefore adversely affecting the accuracy of the S-parameter measurements. The approach in this work is to utilize precision air-dielectric coaxial transmission lines, known as air lines, as primary impedance standards at cryogenic temperatures. Air lines with accurately known dimensions are routinely used as primary impedance standards at RF and microwave frequencies for traceable calibration and measurement applications at room temperature [5].

This work investigates the feasibility of utilizing an air line as a primary impedance standard for cryogenic S-parameter calibrations. Section II describes a calibration scheme suitable for both room temperature and cryogenic temperatures, and the design of an air line in the 3.5 mm coaxial line size for measurements in the frequency range of 2 to 8 GHz. This range covers most of the current applications relating to superconducting quantum computing. The theory and the measurement setup used to characterize the air line at 296 K and 25 mK is described in Section III, along with results showing the air line characterization at both 296 K and 25 mK.

II. CALIBRATION TECHNIQUE AND STANDARDS

A. TRL Calibration Scheme

The National Physical Laboratory has developed PIMMS (Primary IMpedance Measurement System) to provide the UK's national reference for S-parameter measurements [6]. PIMMS strives to achieve the best measurement accuracy from available calibration standards, calibration techniques and instrument hardware. The PIMMS software sends and receives information to and from the VNA and performs the necessary calculations to implement the VNA calibration and measurement algorithms. The measurement strategy makes use of multiple repeated connections to enable the size of the random errors to be determined. PIMMS principally supports the Thru-Reflect-Line (TRL) scheme for calibration of two-port networks. TRL is suitable for use at mK temperatures since the technique only requires precise knowledge of all four S-parameters of the Thru standard, which for a zero-length, insertable Thru can be safely assumed. Other parameters of the calibration standards, such as the offset length of the reflect standards and physical length of the Line standard, only need to be known within a $\frac{1}{4}$ -wavelength to perform an accurate calibration at mK temperatures. This is critical for measurements at mK temperatures where mechanical dimensions are subject to variation due to thermal contraction.

B. TRL Calibration Standards

The TRL calibration requires measurements of two transmission standards and a pair of reflect standards to determine the two-port error correction coefficients. The Thru standard is a zero-length insertable through connection. The Reflect standard can be any device with a high reflection. Precise knowledge of the value of its reflection is not required. However, the technique requires that both Reflects produce the same reflection. In this work, commercial 3.5 mm coaxial male and female offset short standards manufactured by Maury Microwave are used. The 3.5 mm connector is a metrology grade connector that is mechanically compatible with the connector typically found on devices designed for use in the cryogenic environment, i.e., SMA, and its recommended frequency range, as per IEEE 287.1-2021, includes the frequency range of interest in this work.

The Line standard is designed to provide a change in the transmission phase, with respect to the Thru connection, of approximately 90° (i.e., $\frac{1}{4}$ -wavelength) at frequencies around the middle of the frequency range of interest, with calibration failure occurring when the phase is either 0° or 180° . Typically, the phase changes introduced by the Line standard as compared to the Thru are designed to be within 20° and 160° at each frequency across this range. Therefore, a single Line standard manufactured by Maury Microwave has been used, with a nominal length of 10 mm (i.e., $\frac{1}{4}$ -wavelength at 8 GHz when air is the dielectric). Based on its estimated phase changes, such a line will provide satisfactory calibrations over a frequency range of 1.8 to 13.3 GHz when calibrating a metrology grade setup at room temperature. With the range of interest for this work in the cryogenic environment being 2 to 8 GHz, this air line qualifies as a suitable Line standard.

The characteristic impedance, Z_0 , of a lossless air line relates to the air line conductor dimensions a and b as follows:

$$Z_0 \approx 59.939 \log_e \frac{b}{a} \quad (1)$$

where a is the radius of the center conductor and b is the internal radius of the outer conductor. The dimensions are chosen to set the characteristic impedance of the air line to a specific value, the most common being 50 Ω .

III. CHARACTERIZATION OF A COAXIAL AIRLINE

A. Measurement Theory

Beadless air lines are assumed to exhibit near ideal properties at room temperature (i.e., are virtually reflectionless and have very low insertion loss) due to having air as the dielectric, the absence of dielectric support beads, the use of high conductivity materials for fabricating the air line conductors and the precision connectors at both ends of the line. Practical air lines do have a small amount of loss owing to the finite conductivity (or non-zero resistivity) of the metals (e.g., BeCu) used to fabricate the air line's conductors. This leads to a change in the characteristic impedance of the air line, giving rise to a change in the measured S-parameters. In addition, the effect of temperature on the resistivity of such

conductors and, subsequently, on the characteristic impedance of air lines is not precisely known. Therefore, to use an air line as a reference standard at mK temperatures, it is necessary to determine its characteristic impedance (e.g., during a VNA calibration scheme such as TRL [5]) and/or its propagation constant (e.g., if the air line is to be used as a phase reference [7]) at these temperatures, and ensure that there is no significant deviation in these characteristics from the room temperature values.

An air line can be modelled using a distributed circuit involving series resistance R , series inductance L , shunt conductance G and shunt capacitance C , all per unit length of line. The R , L , G and C terms can be determined using the expressions in [9] and are related to characteristic impedance, Z , of the lossy air line and angular frequency (ω) as follows:

$$Z = \sqrt{\frac{R + j\omega L}{G + j\omega C}}. \quad (2)$$

For an air line conductor of non-zero electrical resistivity, it is essential to characterize its resistivity (ρ), first to calculate the R , L , G and C terms, and finally to precisely determine its characteristic impedance. NPL's primary measurement system implementing the TRL algorithm is utilized in this work to estimate the electrical resistivity of the air line at both room and mK temperatures. The TRL calibration routine [5] outputs an experimental determination of the propagation constant (γ_e) of the Line standard (the 10 mm air line) at each frequency as a byproduct of the VNA calibration process. An analytical estimation of the propagation constant (γ_c) is obtained from a function [8] that depends on the frequency ν , air line conductor dimensions a and b , the relative permittivity ϵ_r and relative permeability μ_r of the dielectric and the resistivity ρ . Since air is used as the dielectric in the line, we can use $\epsilon_r \approx 1$ and $\mu_r \approx 1$. Accurate measured values of a and b for the air line at room temperature were obtained using air-gauging measurement methods [9]; these are 1.519 mm and 3.497 mm respectively. The a and b dimensions of the air line at mK temperatures can be analytically estimated using the thermal coefficient of expansion equation

$$\Delta L = \alpha L_0 \Delta T \quad (3)$$

where ΔL is the change in the respective dimension over a temperature change from 296 K to 25 mK, α is the coefficient of thermal expansion of the air line conductor, ΔT is the change in temperature from 296 K to 25 mK and L_0 is the dimension of the air line at 296 K. The air line used in this work is made of Beryllium Copper (BeCu) and features a thin external layer of gold to prevent the surface of the conductors from tarnishing. The α value used is measured value at 4 K in [10] extrapolated to 25 mK. Using (3), the dimensions of a and b at 25 mK are estimated to contract by 0.3% from measured values at 296 K. Then, a root finding procedure based on the bisection method [8] is used at each frequency to determine a value for ρ , as follows

$$|\gamma_c - \gamma_e| = \zeta \quad (4)$$

where ζ sets the accuracy of the bisection procedure and is chosen to be negligibly small. The measurement setup used to determine the resistivity and other air line characteristics at 296 K and 25 mK is discussed in the next section.

2-port Vector Network Analyzer



Fig. 1. S-parameter measurement setup at 296 K with Line and Reflect calibration standards.

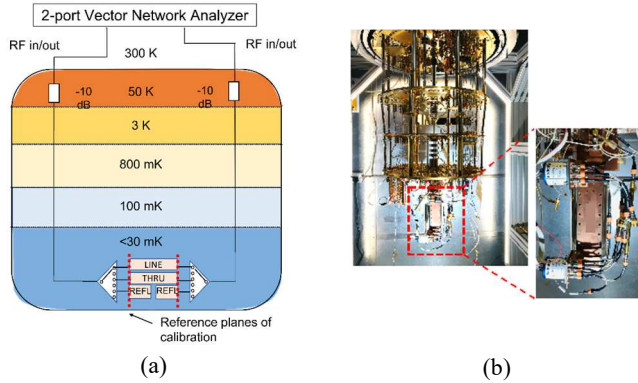


Fig. 2. (a) S-parameter measurement setup at 25 mK; (b) Cryogenic calibration unit with standards inside dilution refrigerator.

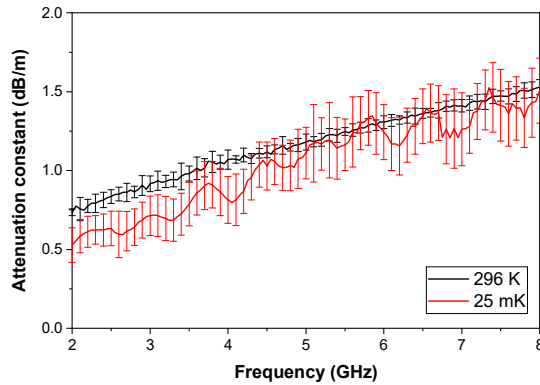


Fig. 3. Attenuation constant of the air line at 296 K and 25 mK.

B. Measurement Setup

The electrical properties of the air line are determined from calibrated S-parameter measurements of the air line. For the measurements at 296 K, a setup using metrology grade cables connected to the VNA was used, as shown in Fig. 1. The mechanical 3.5 mm coaxial calibration standards were physically connected to the cables, and the measurement software was utilized to record the raw data of the standards and perform the VNA calibration.

To characterize the air line at mK temperatures, the S-parameter measurement setup shown in Fig. 2 was used. This setup was adapted from the cryogenic measurement

architectures used in [1]-[4] where cryogenic RF switches were utilized to connect to the calibration standards to record uncalibrated S-parameters. A set of phase-matched coaxial cables manufactured by Maury Microwave were used to connect the standards to the RF switch ports. The cryogenic calibration unit housing the calibration standards and RF switches was deployed in the mixing chamber (<25 mK) of the dilution refrigerator. This architecture is preferred over one that requires making mechanical connections of the calibration standards across different cooling cycles as it considerably reduces both random and drift errors.

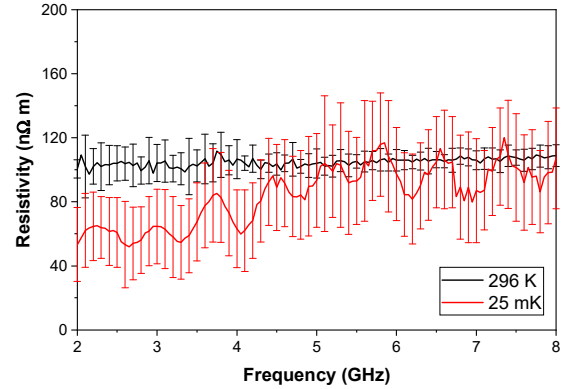


Fig. 4. Resistivity of the air line conductors at 296 K and 25 mK.

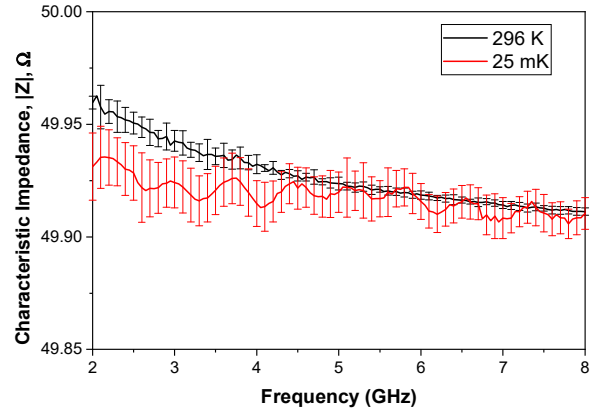


Fig. 5. Magnitude of the characteristic impedance of the air line at 296 K and 25 mK.

C. Measurement Results and Discussion

The calibration process was performed at both 296 K and 25 mK using the measurement setups discussed in the previous section. Five sets of repeated S-parameter measurements were made at 296 K and 25 mK to determine the experimental variability of the air line characteristics. The experimental standard deviation, at each frequency, has been used to represent the standard uncertainty in the determined air line characteristics.

The attenuation constant of the air line was obtained at 296 K and 25 mK as a byproduct of the TRL calibration. This is plotted in Fig. 3. The measured attenuation at 25 mK is lower than at 296 K, expected to be due to the reduced resistivity of the air line conductors at 25 mK, as shown in Fig. 4. To help summarize these values, the resistivity values at all

frequencies were averaged to give 104 nΩm at 296 K and 82 nΩm at 25 mK, which corresponds to a reduction in resistivity of around 20% for the air line's conductors for a change in temperature from 296 K to 25 mK. The determined resistivity at 296 K agrees closely with previous determinations of similar air lines made of BeCu [7]. The determined resistivity at 25 mK is comparable to the reported resistivity of 82 nΩm at 4 K [10], with the newly determined results indicating a limited change in the resistivity of BeCu at lower than 4 K.

The characteristic impedance of the air line at 296 K and 25 mK is determined using (2), and its magnitude is plotted in Fig. 5 as a function of frequency. These results show that there is no significant change in characteristic impedance of the air line at 25 mK, demonstrating that air lines such as this are suitable as reference impedance standards for S-parameter calibrations at cryogenic temperatures down to tens of mK.

The S_{21} phase measurements of the air line and phase constants obtained from the TRL calibration are used to determine the electrical length of the air line at 296 K and 25 mK. The mechanical length of the air line is determined through traceable dimensional measurements at 293 K. This value is then analytically extrapolated using (3) to estimate the air line length at 25 mK. Each of these length determinations are plotted in Fig. 6. A contraction of 0.3% is observed in the electrically determined length at 25 mK compared to that at 296 K. The analytically extrapolated length of the air line at 25 mK gives a contraction of 0.36% compared to the mechanical measurement at 293 K. This indicates that the assumed coefficient of thermal contraction gives a slightly over-estimated value when used over a large ΔT . Although the thermal coefficient of expansion/contraction is a non-linear function of temperature, in this case (3) provides a linear, first order approximation. In any case, these levels of contraction equate to a small fraction of a wavelength at these frequencies, and therefore are of sufficiently small magnitude as to have negligible effect on the performance of the air line as an effective Line standard.

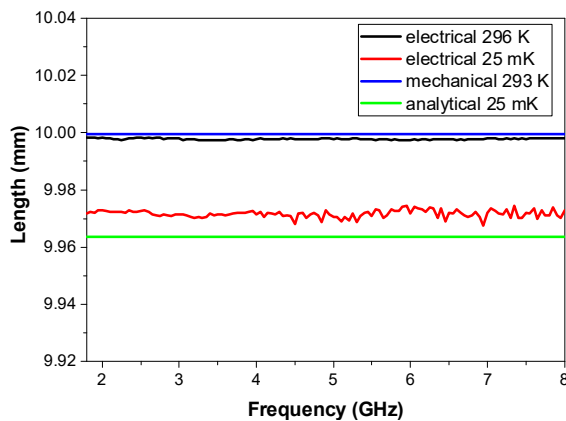


Fig. 6. Determinations of the air line length at 296 K and 25 mK.

In principle, there are likely to be several sources of error that contribute to the uncertainty of these air line characterization results at 25 mK. A key example is the non-identical performance of the RF switch ports and the cables

connecting the standards to the VNA ports, as reported in [1]. These form non-identical paths to each of the standards, introducing reference plane errors in the calibration. Currently, these systematic errors are under investigation at NPL.

IV. CONCLUSION

This work has investigated the feasibility of using air lines as reference standards for S-parameter measurements at mK temperatures. Various measurement techniques have been presented to experimentally determine the characteristics of a 10 mm air line, such as propagation constant, electrical resistivity, and characteristic impedance, at both 296 K and 25 mK. A reduction in measured attenuation and resistivity was observed at 25 mK compared to that at 296 K. No significant deviation in characteristic impedance of the air line was observed at 25 mK compared to that at 296 K. Thermal contraction and the resultant change in air line length was assessed at mK. The results of this investigation indicate that such air lines are suitable for use as primary impedance standards at mK temperatures, and so these air lines can be used to establish traceability to the International System of Units (SI) for S-parameter measurements made at cryogenic temperatures.

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