On Searching All Solutions of Microwave Filter Synthesis Based on Interval Arithmetic

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***Abstract*— In this letter, the numerical interval Newton algorithm (NINA) is introduced into coupling matrix (CM) synthesis (reconfiguration) for the first time. In particular, all existing real solutions to microwave filter CM can be found with mathematical certainty, which provides designers with the flexibility to select the most suitable configuration. Also, CM limits can be directly imposed in the synthesis process to only compute solutions in the specified range. This synthesis process is independent of initial variable values, and the global convergence is theoretically guaranteed. Moreover, NINA is a deterministic method that can verify the existence or nonexistence, uniqueness or non-uniqueness of a given CM topology with the desired filtering performance. Furthermore, the processes dealing with both well-determined and over-determined systems are discussed to cover the synthesis of CMs with diverse configurations. One practical sixth-order dielectric filter with parasitic couplings is exemplified to validate the effectiveness of the presented synthesis approach.**

***Index Terms*— All solutions, coupling matrix (CM), filter synthesis, interval arithmetic.**

I. Introduction

THERE are increasingly stringent requirements on microwave filters in modern terrestrial and space-based communication systems, such as lightweight and compact size. Consequently, parasitic couplings become inevitable in extremely limited space, and filter topologies are requested to be diversified to adapt to layout constraints. Therefore, the synthesis (reconfiguration) of filters with irregular topologies becomes necessary.

The analytical synthesis based on the specific sequence of similarity transformations is well-known [1], featuring very high efficiency. However, sometimes it may be difficult or even impossible to derive the sequence of matrix rotations via mere observation for each given topology. Hence, general optimization-based synthesis has gained widespread popularity [2], [3], [4], [5] in practice, but local optimizations rely much on initial values and easily fall into local minimums. To tackle this issue, global optimization algorithms were employed in [6] and [7]. Besides, numerical processes based on homotopy continuation [8] and Remez-like iterations [9] were presented to solve certain non-canonical configurations. These optimization and numerical approaches discussed above aim at obtaining only one solution, unless the variables are randomly initialized many times. Nevertheless, finding all the solutions are desired in many cases to help designers select the one that best fits the implementation technology and physical layout. Fortunately, the Groebner basis was exploited to exhaustively derive all the synthesis solutions in [10]. Whereas the mathematical details are not fully disclosed. This method has been embedded in a software package [11], but it is not easy to customize some target topologies, especially when adding limits to only synthesize solutions in a prescribed range.

In this letter, the numerical interval Newton algorithm (NINA) [12] is applied to microwave filter coupling matrix (CM) synthesis for the first time. This is a rather simple and straightforward mathematical process that is fully disclosed and can be easily implemented using the platform INTLAB [13] developed for MATLAB [14]. Besides, the computation results globally converge without the need for initial guesses for variables [15]. Most importantly, NINA finds **ALL** solutions and can prove the existence or nonexistence, uniqueness or non-uniqueness of a specified filter topology [16], [17]. Thereby, designers can determine the feasibility of the target configuration and select the most suitable solution. Also, NINA can proceed with added limits to only synthesize solutions in a prescribed range, thus finding the most practical one with the least time. To accommodate more topologies, the process of dealing with well-determined and over-determined equations using NINA is discussed. Finally, a sixth-order dielectric filter with parasitic coupling values limited in a preset range is used to verify the effectiveness of the presented method.

II. Interval Arithmetic

A. Interval Number System

Interval arithmetic is a well-established mathematical field [16], in which the values participating in the computation are represented with closed intervals instead of single values. For example, an interval variable can be denoted as the capital letter $X = [\underline{X}, X]$. Here, $\underline{X}$ and $\bar{X}$ are the left and right endpoints of X, respectively. When the width of X (i.e., X–$\underline{X}$) is smaller than the preset error, $x = (\underline{X} + X)/2$ can be regarded as the solution.

The operation rules of intervals are well documented in [17], and both vectors and matrices can be in the interval form. All these basic arithmetic rules have been defined in INTLAB.

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B. Interval Newton Method

The NINA algorithm, which can be viewed as a combination of Newton's method and interval arithmetic, has been widely applied in many other fields, such as rounding error analysis and computer-assisted proofs [17]. It is a powerful tool to compute all real roots of nonlinear equations of the form

$$F(x) = \begin{cases} f_1(x_1, \ldots, x_n) = 0 \\ f_2(x_1, \ldots, x_n) = 0 \\ \vdots \\ f_n(x_1, \ldots, x_n) = 0. \end{cases} \tag{1}$$

The process of NINA is like gradually contracting the wide interval ranges of variables to final convergent solutions. The starting n-dimensional real interval vector should be initialized as $X^0 = [\underline{X}_1, \bar{X}_1] \times \cdots \times [\underline{X}_n, \bar{X}_n]$. Modifying single numbers in the conventional Newton method to intervals, the iterative approximation of the root interval is

$$K\left(X^i\right) = y^i - F'\left(X^i\right)^{-1} \cdot F\left(y^i\right) \tag{2}$$

where X^i is the interval vector in the ith iteration and y^i is a vector that is in the set of X^i. Usually, y^i is set as the midpoint of X^i, i.e., $y^i = \mathrm{mid}(X^i)$. $F'(X^i)$ denotes the interval extension [17] of the Jacobian matrix $F'(x)$ over X^i. The Krawczyk method [12] can be used to approach the term $F'(X^i)^{-1}$, and then (2) can be rewritten as

$$\begin{aligned} K\left(X^i\right) &= y^i - Y \cdot F\left(y^i\right) + \left\{I - Y \cdot F'\left(X^i\right)\right\}\left(X - y^i\right) \\ Y &= \mathrm{mid}\left(F'\left(X^i\right)\right)^{-1}. \end{aligned} \tag{3}$$

Comparing $K(X^i)$ and X^i, here are three cases [17] as follows.

1) If $X^i \cap K(X^i) = \text{empty}$, there is no solution in X^i, and it can be pruned.

2) If $K(X^i) \subseteq X^i$, there is a unique solution in X^i, and push X^i into the solution list *Sol*.

3) Otherwise, there is no conclusion. When $K(X^i)$ exists, update the interval vector as $X^{i+1} = K(X^i) \cap X^i$. If the width of X^{i+1} is smaller than the preset error value (such as $1e-3$), push it into *Sol*. If not, send X^{i+1} into the next cycle to compute the corresponding $K(X^{i+1})$ using (3). When $K(X^i)$ is not defined, bisect X^i from the widest variable interval into $X^{i+1,1}$ and $X^{i+1,2}$, and deliver them into the next iteration.

For each interval, repeat the above cycles until the interval list is empty, and then the midpoints of the intervals in *Sol* are the roots of the equation system. If there is no interval in *Sol*, the equation system has no solution. The convergence of this process is mathematically guaranteed [15]. When NINA is applied to topology synthesis, to make (3) proceed properly, the constructed mathematical model should be well-determined (discussed later in Section III), and its corresponding Jacobian matrix needs to be full rank. If the assigned target topology is of no solution, all the subdivided interval vectors will be pruned from the queue finally according to case 1); thus, the empty list *Sol* is returned. The flowchart of the IN algorithm is depicted in Fig. 1. In this process, the branch process helps subdivide the intervals, while the Krawczyk operator determines the existence of roots in each interval and decides whether to further contract or prune the current interval.

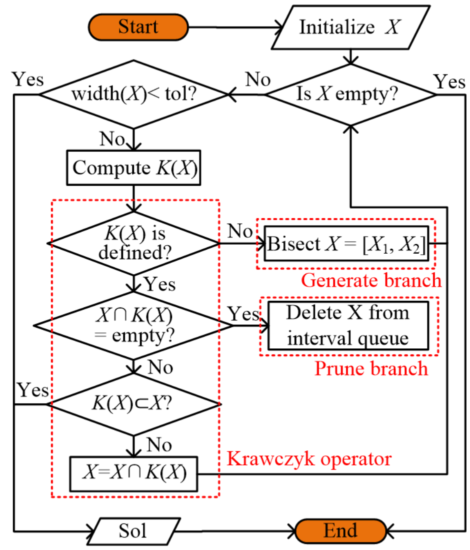

Fig. 1. Flowchart of the NINA algorithm.

III. Synthesis With Interval Newton Method

A. Well-Determined Systems

In well-determined systems, the number of variables is exactly equal to the number of equations. Take a fifth-order filter with the return loss RL = 20 dB and transmission zeros (TZs) $\omega_{\mathrm{tz}} = [2.4, -1.6, -3]$ as an example. The target filter topology is named "quintet-like," which is not included in [11], and the three solutions calculated with NINA are displayed in Fig. 2(a)–(c). Here, the equation system $M_T = Q^T M_c Q$ is established based on similarity transforms [10], where Q is the unknown orthogonal transform matrix, M_c is the canonical CM in the folded form, and M_T represents the CM in the target quintet-like form. Q is in the form of

$$Q = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & Q' & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad Q' = \begin{pmatrix} x_1 & x_2 & x_3 \\ x_4 & x_5 & x_6 \\ x_7 & x_8 & x_9 \end{pmatrix}. \tag{4}$$

The nonlinear equations $F(x)$ in (1) consist of

$$\begin{aligned} &\forall (i, j) \in T, \left(Q^T M_c Q\right)_{ij} = 0 \\ &Q^T Q = I \end{aligned} \tag{5}$$

where T contains the indices of the couplings that should be zero in a target topology. In this quintet-like example, the equations are

$$\begin{aligned} &M_{T,13} = 0, M_{T,24} = 0, M_{T,34} = 0 \\ &\left(Q^T Q\right)_{3,4} = 0, \left(Q^T Q\right)_{3,5} = 0, \left(Q^T Q\right)_{4,5} = 0 \\ &\left(Q^T Q\right)_{3,3} = 1, \left(Q^T Q\right)_{4,4} = 1, \left(Q^T Q\right)_{5,5} = 1. \end{aligned} \tag{6}$$

Initialize the variable interval as $X = [-1, 1] \times \cdots [-1, 1]$. Solving (6) by following the procedures listed in Fig. 1, the three solutions are obtained after 32 iterations (the coupling sign symmetry has been removed [10]). There are 936 intervals at most in the iterative process. The calculation based on MATLAB takes 66.8 s on an Intel Core i7 processor at 2.90 GHz. With the further acceleration of this algorithm in the future, the computation time can be considerably shortened.

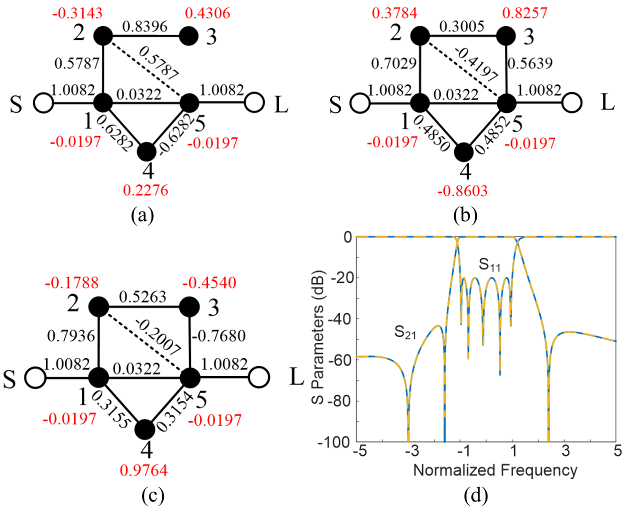

Fig. 2. (a)–(c) Three synthesis solutions of the fifth-order quintet-like example. The black numbers denote intercoupling values, while the red numbers represent self-coupling values. (d) Synthesized responses of the folded form CM and the obtained CM in the extended box form. Dashed line denotes the folded form CM. Solid line denotes the extended box CM.

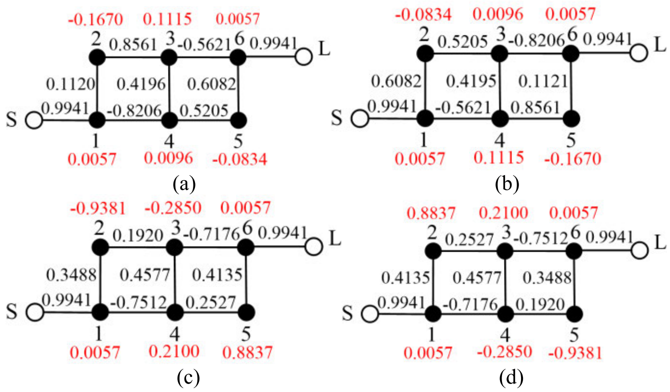

Fig. 3. (a)–(d) Four synthesis solutions of the sixth-order extended box example. The black numbers denote intercoupling values, while the red numbers represent self-coupling values.

It can be observed that M_{35} equals zero in Fig. 2(a), which forms a simplified topology. Their responses in the normalized lowpass domain shown in Fig. 2(d) are the same as that of M_c, which has validated these solutions.

B. Over-Determined System

Generally speaking, the standard NINA algorithm is aimed to solve well-determined systems. Nevertheless, the equation systems in CM synthesis are not always well-determined but are sometimes over-determined (the number of equations is more than the number of variables), which brings non-square Jacobian matrices. In this situation, the equation system should be "squared" using a novel approach with minimum effort.

Take a sixth-order filter with RL $= 20$ dB and $\omega_{\text{tz}} = [1.25, -1.5]$ as an instance. The target topology is the extended box. Finally, four real solutions are derived (coupling sign symmetry has been removed), and the corresponding CMs are shown in Fig. 3. The variable vector is $x = [x_1, x_2, \ldots, x_{16}]$. Whereas, the equation system has 17 equations

$$\begin{gathered}M_{T,13} = 0,\ M_{T,15} = 0,\ M_{T,24} = 0,\ M_{T,25} = 0,\ M_{T,26} = 0\\ M_{T,35} = 0,\ M_{T,46} = 0,\ \left(Q^TQ\right)_{3,4} = 0,\ \left(Q^TQ\right)_{3,5} = 0\\ \left(Q^TQ\right)_{3,6} = 0,\ \left(Q^TQ\right)_{4,5} = 0,\ \left(Q^TQ\right)_{4,6} = 0,\ \left(Q^TQ\right)_{5,6} = 0\\ \left(Q^TQ\right)_{3,3} = 1,\ \left(Q^TQ\right)_{4,4} = 1,\ \left(Q^TQ\right)_{5,5} = 1,\ \left(Q^TQ\right)_{6,6} = 1.\end{gathered} \tag{7}$$

Hence, an additional variable x_{17} is added and its interval is imposed to be [0, 0]. This means the variable interval is initialized as $X = [-1, 1] \times \cdots [-1, 1]$. Then, the extra variable can be added onto one of the equations, such as the eighth equation in (7), to form (7′)

$$\left(Q^TQ\right)_{3,4} + x_{13} = 0. \tag{8}$$

In this way, the NINA algorithm can proceed normally, and the roots of (7′) are exactly the solutions of (7). Similarly, 73 iterations are performed in total, and the number of intervals has reached a peak of 986 in the computation process. In [11], the reference files with precalculating information specific to given topologies are necessary, which greatly saves the time spent on time-consuming symbolic computation [4]. NINA can be accelerated similarly in future work. The synthesized responses of the folded form CM and the obtained extended box CMs are compared in Fig. 4.

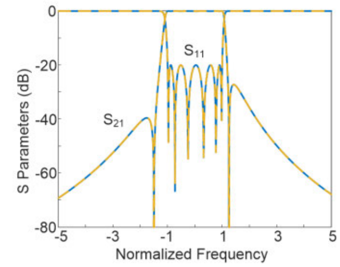

Fig. 4. Synthesized responses of the folded form CM and the obtained CM in the extended box form. Dashed line denotes the folded form CM. Solid line denotes the extended box CM.

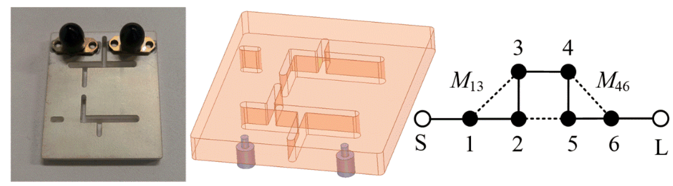

Fig. 5. Fabricated sixth-order dielectric-filled waveguide filter with two TZs.

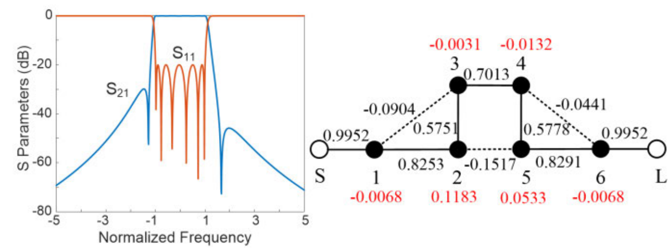

Fig. 6. Synthesized golden circuit and the response of the dielectric filter example.

IV. Experimental Example of Synthesis With Limits

Besides computing all circuit solutions, NINA can directly synthesize with customized limits to derive solutions in a prescribed range, thus best fitting the reality, which is not supported by Dedale-HF. Fig. 5 displays a manufactured sixth-order dielectric-filled waveguide filter with two TZs and its

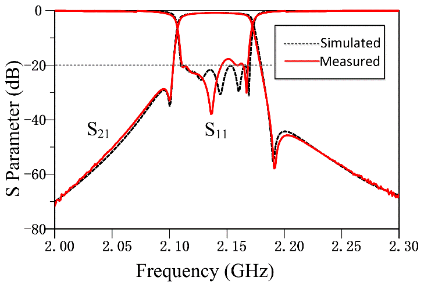

Fig. 7. Simulated and measured responses of the dielectric filter example.

coupling diagram. To facilitate fabrication and enhance structure strength, blind holes are avoided in this design. To realize the asynchronously tuned resonances but still retain the regular rectangular shape without irregular edges, the lengths and widths of all resonant cavities are deliberately adapted to each other. In this case, however, the parasitic couplings M_{13} and M_{46} are unavoidable, which causes the asymmetry of TZs on both sides of the passband. Their equivalent values can be extracted from EM simulations with lumped ports [18]. Although this topology has multisolutions, only the CM with M_{13}, $M_{46} \in [-0.1, 0]$ is expected since the extracted parasitic couplings are in this interval. Fortunately, with the NINA algorithm, it is convenient to limit some coupling values inside specified ranges via properly initializing the variable intervals. Set RL $=$ 20 dB and $\omega_{\mathrm{tz}} = [1.654, -1.284]$. The synthesized golden circuit with suitable M_{13} and M_{46}, and the corresponding responses are presented in Fig. 6, which can guide the design and tuning process. The filter is dimensioned in a conventional way, and the final simulation and measured results are displayed in Fig. 7.

V. Conclusion

In this letter, NINA is applied in microwave filter synthesis for the first time to reliably find all solutions or solutions in a specified range. To handle various topologies, the way to process well-determined and over-determined systems is discussed. In our future work, the acceleration of NINA will be explored and the synthesis of other filter configurations, like frequency-dependent couplings or non-resonating nodes, may be included.

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