Student Paper Finalist

WE01E - 1

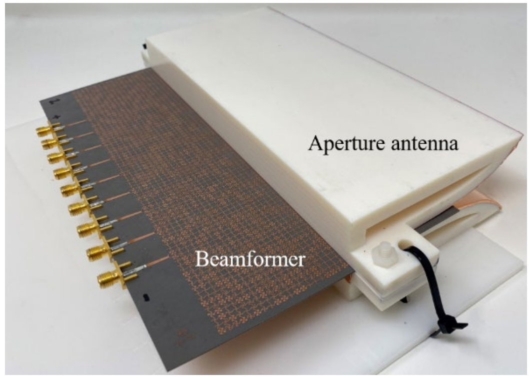
Inverse Design of Perfectly-Matched Metamaterials

S. Thakkar[1], L. Szymanski[1], J. Ruiz-Garcia[1], G. Gok[2], and A. Grbic[1]

[1]Department of Electrical Engineering and Computer Science, University of Michigan, USA

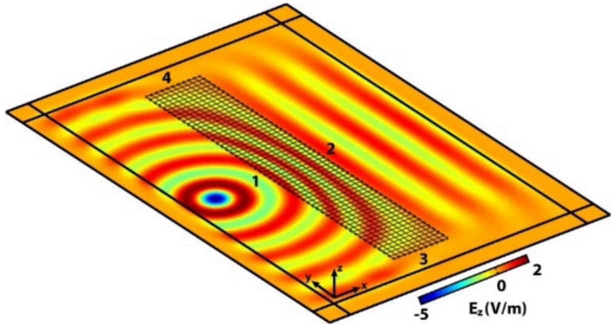
[2]Raytheon Technologies Research Center, East Hartford, USA

Motivation



[1]

- Inverse design allows the design of complex metasurfaces having high-dimensional design spaces
- Inverse-designed structures often exploit standing waves within the device to achieve a desired function [1]
- This can limit bandwidth and contribute to loss



[2]

- [2] produces a collimated beam with triangular power density from a line source excitation
- The device is discretized into magnetically anisotropic blocks defined by their wave and Poynting vectors.
- The unit cell permittivities were optimized to reduce inter-cell reflections
- Impractical to extend this procedure to MIMO devices.

This work

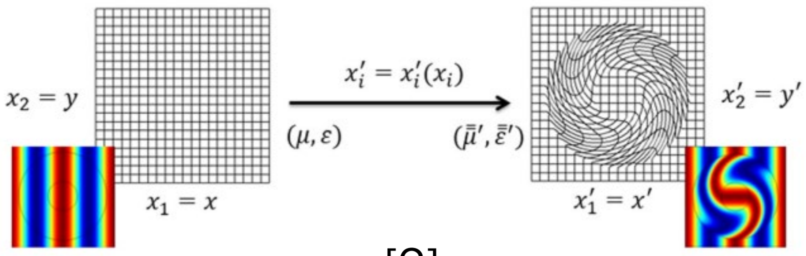
- Utilize unit cells that are impedance matched to the surrounding and each other under all possible excitations [3] with an inverse design procedure
- Enables the design of true time-delay devices with broadband operation
- The perfectly-matched metamaterial (PMM) transforms a point source excitation into a collimated beam with trapezoidal amplitude taper

[1] L. Szymanski, G. Gok, and A. Grbic, *IEEE Antennas Propag. Mag.*, vol. 64, no. 4, pp. 63–72, Aug. 2022
[2] G. Gok and A. Grbic, *Phys. Rev. Lett.*, vol. 111, no. 23, p. 233904, Dec. 2013
[3] G. Gok and A. Grbic, *J. Opt.*, vol. 18, no. 4, p. 044020, Apr. 2016

Background

Transformation Optics

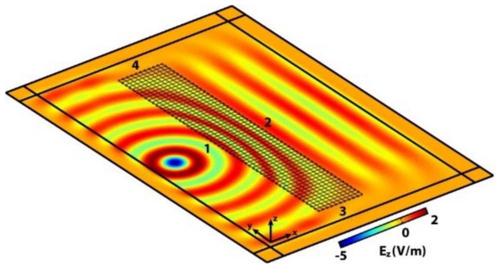
- A desired field distribution can be achieved through the stretching/compression of space
- The transformed medium is anisotropic, inhomogeneous and impedance matched to the surrounding medium and devoid of internal reflections



[2]

Alternate Approach

- In practice, discretizing the transformed medium deteriorates the performance.
- Alternatively, in [1], the medium is discretized into magnetically anisotropic blocks such that the determinant of the permeability is equal to that of the surroundings

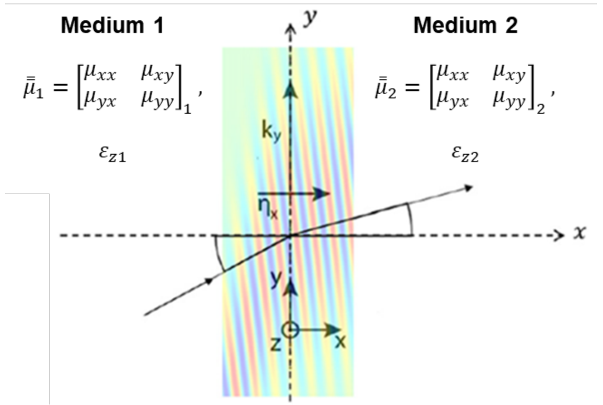


[1]

All-Angle Reflectionless Media

- [2] states the following condition for two media to be impedance-matched under all excitations

$$\boxed{\begin{gathered}|\bar{\bar{\mu}}|_1 = |\bar{\bar{\mu}}|_2 = \Delta, \\ (\mu_{xx}\varepsilon_z)_1 = (\mu_{xx}\varepsilon_z)_2 = \kappa\end{gathered}}$$

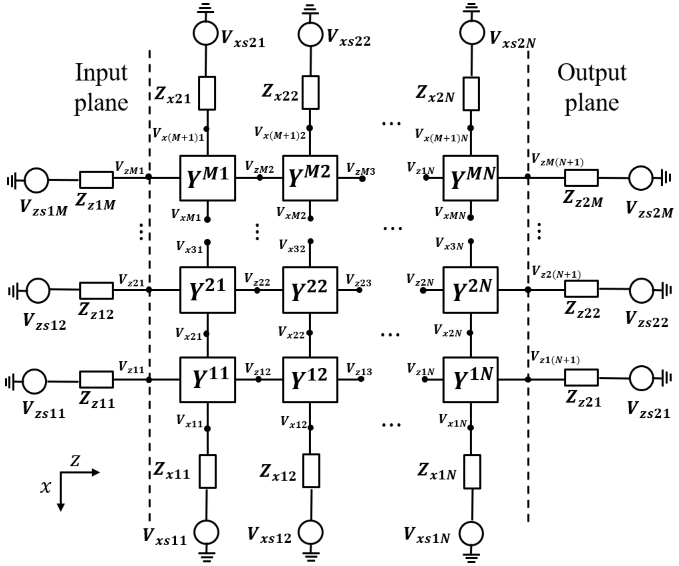


[1]

[1] G. Gok and A. Grbic, *Phys. Rev. Lett.*, vol. 111, no. 23, p. 233904, Dec. 2013
[2] G. Gok and A. Grbic, *J. Opt.*, vol. 18, no. 4, p. 044020, Apr. 2016

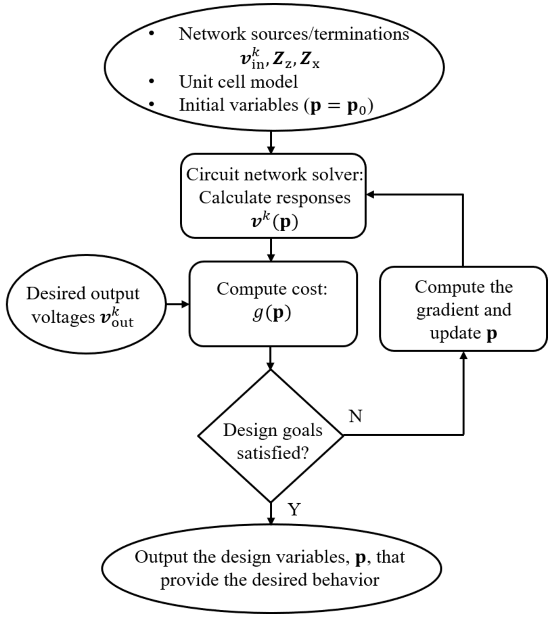
Inverse-Design Procedure

- The all-angle impedance-matched unit cells are integrated into an inverse design routine [1], described here.
- The device is discretized into unit cells that are represented by their Y-matrices
- Voltages in the grid are solved by imposing KCL at every node, referred to as the 2D circuit network solver.



[1]

- The optimization routine employs a gradient-descent-based method to minimize the cost function, based on the difference between the solved voltages and desired voltages
- The routine is accelerated by the adjoint variable method



Optimization routine [1]

[1] L. Szymanski, G. Gok and A. Grbic, *IEEE Trans. Antennas Propag.*, Dec. 2021.

Perfectly Matched Media

Consider a magnetically anisotropic medium under an S-polarized (TE_z) excitation. Therefore

$$\bar{E} = E_z\hat{z}$$
$$\bar{H} = H_x\hat{x} + H_y\hat{y}$$

We can write down the time-harmonic Maxwell's equations as

$$\begin{bmatrix}\overline{k_x}\\ \overline{k_y}\end{bmatrix} = |\bar{\bar{\mu}}|\begin{bmatrix}\mu_{xx} & \mu_{xy}\\ \mu_{yx} & \mu_{yy}\end{bmatrix}^{-1}\begin{bmatrix}\overline{\eta_x}^{-1}\\ \overline{\eta_y}^{-1}\end{bmatrix}$$
$$\varepsilon_z = \begin{bmatrix}\overline{k_x} & \overline{k_y}\end{bmatrix}\begin{bmatrix}\overline{\eta_x}^{-1}\\ \overline{\eta_y}^{-1}\end{bmatrix}$$

where $\overline{\eta_x} = -\frac{1}{\eta_0}\frac{E_z}{H_y}$, $\overline{\eta_y} = \frac{1}{\eta_0}\frac{E_z}{H_x}$ are the normalized wave impedances

Let, $\overline{k_x} = \frac{k_x}{k_0}$ and $\overline{k_y} = \frac{k_y}{k_0}$ be the normalized wave numbers and $|\bar{\bar{\mu}}|$ the determinant of the permeability tensor

Assuming the permeability tensor is symmetric, the dispersion equation becomes:

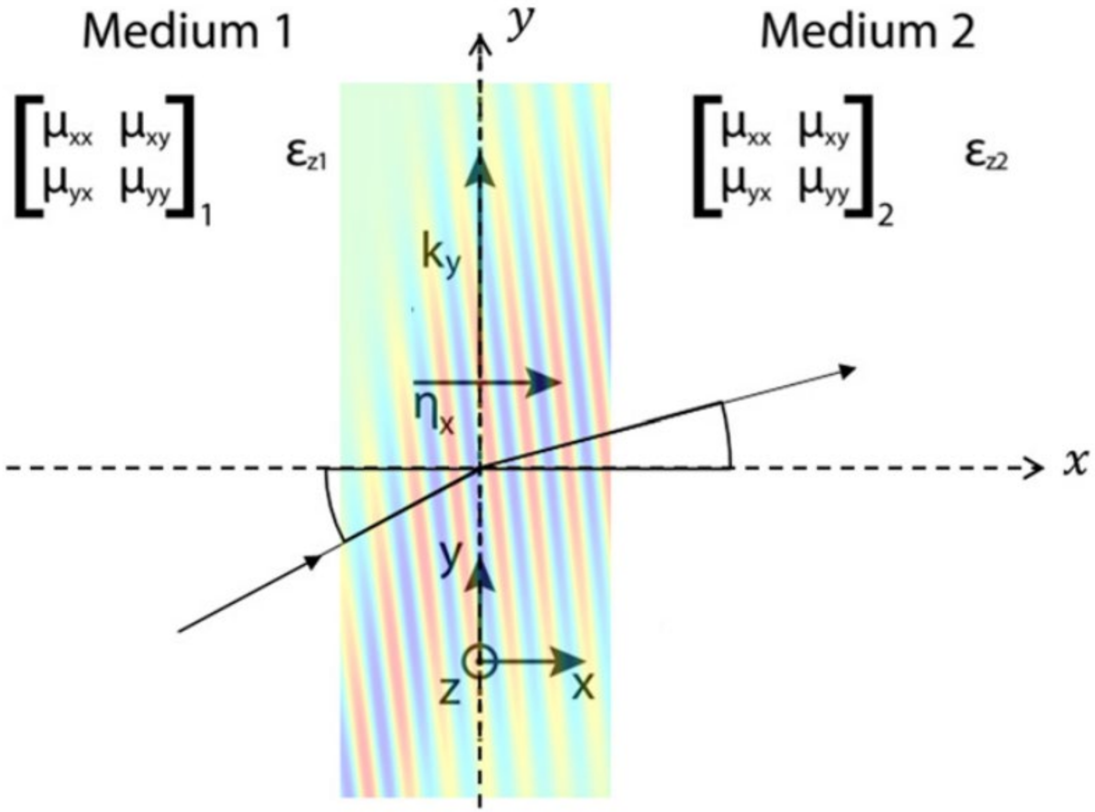
$$\overline{k_x}^2\frac{\mu_{xx}}{|\bar{\bar{\mu}}|} + 2\overline{k_x}\,\overline{k_y}\frac{\mu_{xy}}{|\bar{\bar{\mu}}|} + \overline{k_y}^2\frac{\mu_{yy}}{|\bar{\bar{\mu}}|} = \varepsilon_z$$

The normalized wave impedances can be written as

$$\overline{\eta_x}^{-1} = \overline{k_x}\frac{\mu_{xx}}{|\bar{\bar{\mu}}|} + \overline{k_y}\frac{\mu_{xy}}{|\bar{\bar{\mu}}|},$$
$$\overline{\eta_y}^{-1} = \overline{k_x}\frac{\mu_{yx}}{|\bar{\bar{\mu}}|} + \overline{k_y}\frac{\mu_{yy}}{|\bar{\bar{\mu}}|}$$

Perfectly-Matched Media

Consider an interface aligned with the y axis, separating two homogeneous, magnetically anisotropic media



For the two media to be impedance matched under all excitations, it is necessary that for all values of $\overline{k_y}$

$$(\overline{\eta_x})_1 = (\overline{\eta_x})_2$$

We can relate $\overline{k_y}$ and $\overline{\eta_x}$ using the dispersion relation and wave impedance expression as follows

$$\frac{\left(\frac{1}{\overline{\overline{\eta_x}}}\right)^2}{\left(\sqrt{\frac{\mu_{xx}\varepsilon_z}{|\bar{\bar{\mu}}|}}\right)^2} + \frac{\overline{k_y}^2}{\left(\sqrt{\mu_{xx}\varepsilon_z}\right)^2} = 1$$

If $\left(\overline{k_y}\right)_1 = \left(\overline{k_y}\right)_2$ and $(\overline{\eta_x})_1 = (\overline{\eta_x})_2$, the conditions for all-angle impedance matching can be derived from comparing the equation above for the incident and refracted wave. They are

$$\boxed{\begin{gathered} |\bar{\bar{\mu}}|_1 = |\bar{\bar{\mu}}|_2 = \Delta, \\ (\mu_{xx}\varepsilon_z)_1 = (\mu_{xx}\varepsilon_z)_2 = \kappa \end{gathered}}$$

Perfectly-Matched Media

The conditions for all-angle impedance matching are

$$\boxed{\begin{gathered} |\bar{\bar{\mu}}|_1 = |\bar{\bar{\mu}}|_2 = \Delta, \\ (\mu_{xx}\varepsilon_z)_1 = (\mu_{xx}\varepsilon_z)_2 = \kappa \end{gathered}}$$

where,

Δ represents the determinant of the permeability tensor of the surrounding media,

κ represents the normal index of refraction in the media

NOTE: If the medium is surrounded by free space,

$$\Delta = \kappa = 1.$$

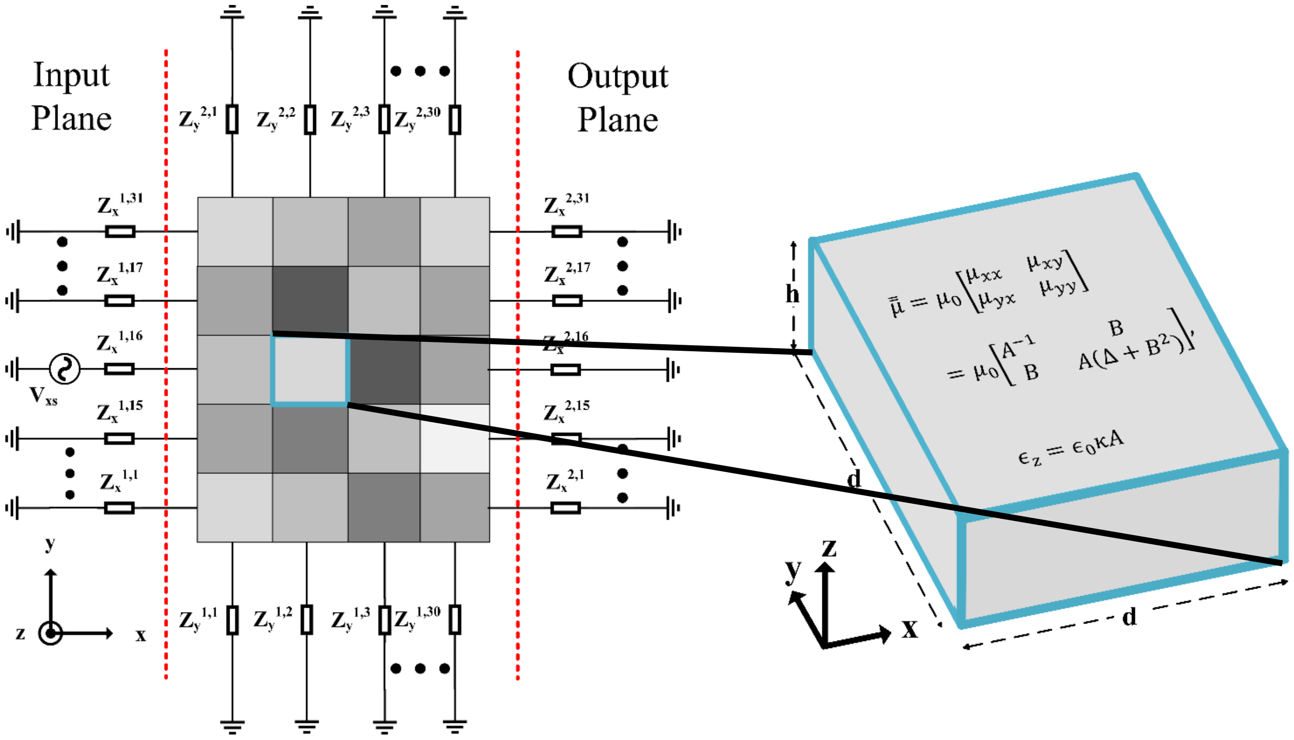
Equivalently, the material parameters of the perfectly-matched medium can be written as

$$\boxed{\bar{\bar{\mu}} = \begin{bmatrix} A^{-1} & B \\ B & A(\Delta + B^2) \end{bmatrix}, \varepsilon_z = \kappa A}$$

This ensures that $|\bar{\bar{\mu}}| = 1$ and $\mu_{xx}\varepsilon_z = \kappa$.

Therefore, we have two free variables, A and B, that define the properties of the medium.

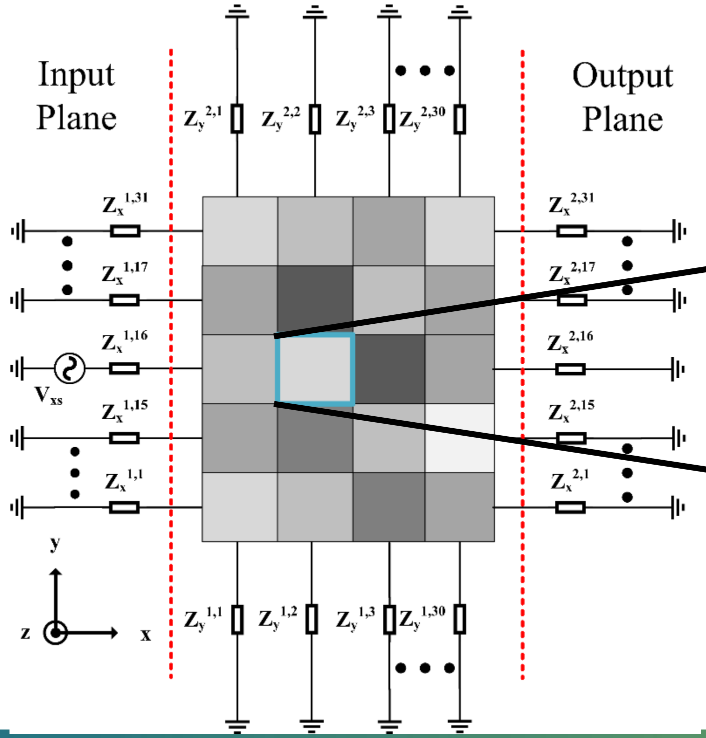
We propose to divide our computational domain into homogenous blocks of perfectly-matched media (PMM) as shown below and optimize A and B of each unit cell to achieve a desired function.

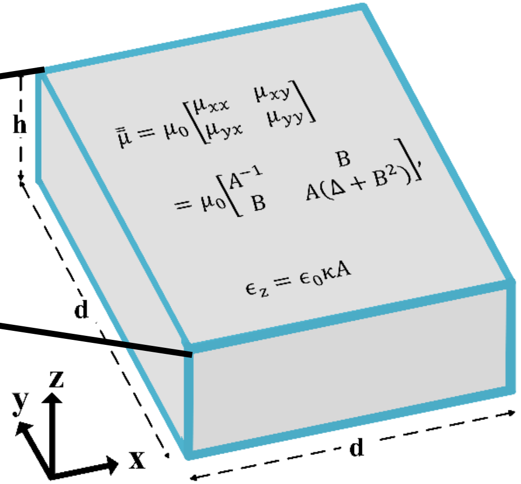


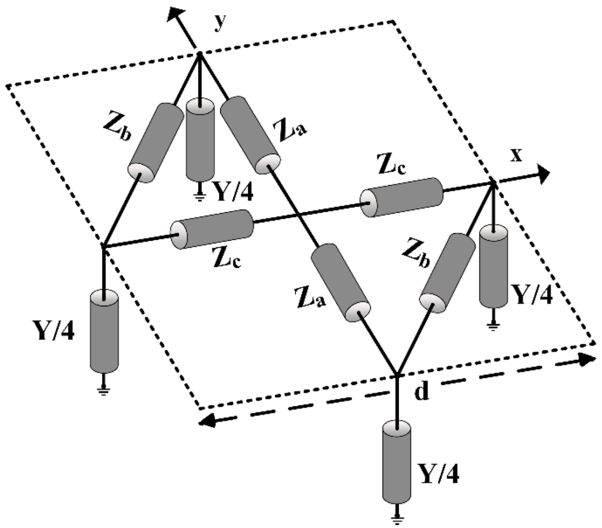
Equivalent Transmission-Line Circuit

- The 2D circuit network solver computes the forward solution using the Y-matrix of each unit cell and imposing KCL at nodes between unit cells.
- An equivalent circuit representation, and hence the required Y-matrix, is derived for a perfectly-matched media as described previously

- A tensor transmission-line unit cell of shunt node configuration and bowtie topology is shown below
- Z_c and Z_a are the series impedances along the *x* and *y* axes
- Z_b is the series impedance along the *x*-*y* diagonal
- Y is the shunt admittance of the unit cell

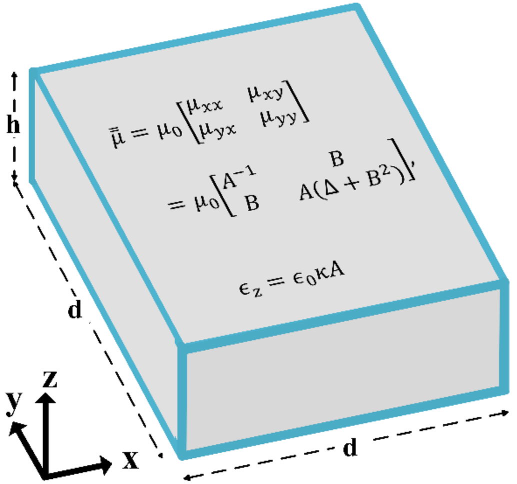




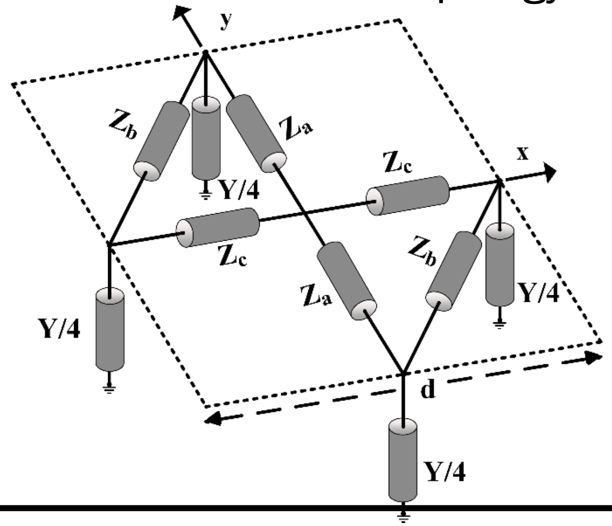


Equivalent TLine Circuit

Homogenized Anisotropic block



Bowtie Circuit Topology



$$\bar{\bar{\mu}} = \begin{bmatrix} A^{-1} & B \\ B & A(\Delta + B^2) \end{bmatrix},$$

$$\varepsilon_z = \kappa A$$

$$\bar{\bar{Z}} = \begin{bmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{bmatrix} = \begin{bmatrix} \dfrac{2Z_c(Z_a + Z_b)}{Z_a + Z_b + Z_c} & -\dfrac{2Z_a Z_c}{Z_a + Z_b + Z_c} \\ -\dfrac{2Z_a Z_c}{Z_a + Z_b + Z_c} & \dfrac{2Z_a(Z_c + Z_b)}{Z_a + Z_b + Z_c} \end{bmatrix},$$

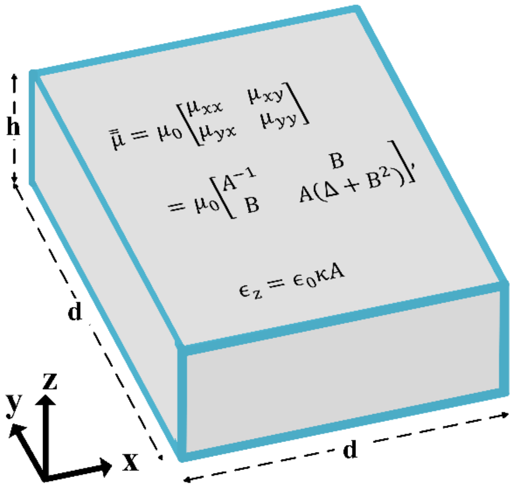
$$Y$$

A one-to-one relationship between material and circuit parameters can be established as follows

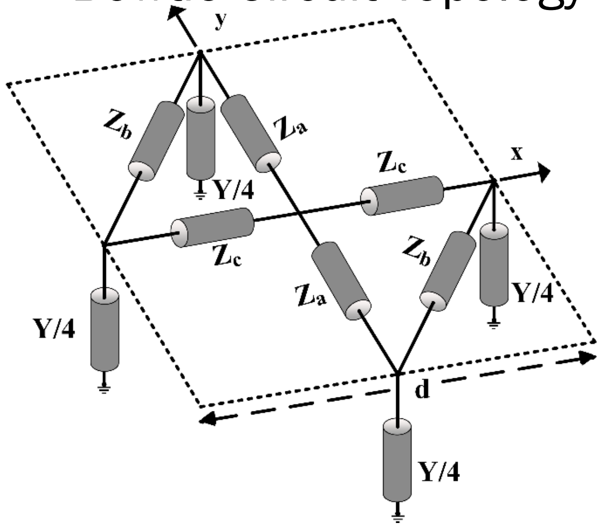
$$j\omega d \begin{bmatrix} \mu_{yy} & -\mu_{xy} \\ -\mu_{yx} & \mu_{xx} \end{bmatrix} = \begin{bmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{bmatrix}, \; j\omega d \epsilon_z = Y$$

Equivalent TLine Circuit

Homogenized Anisotropic block



Bowtie Circuit Topology



Hence, the circuit parameters can be represented in terms of A and B in the following way

$$Z_a = \frac{j\omega d\Delta}{2[A(\Delta + B^2) - B]}$$

$$Z_b = \frac{j\omega d\Delta}{2B}$$

$$Z_c = \frac{j\omega d A\Delta}{2(1 - AB)}$$

$$Y = j\omega d\kappa A$$

Note that for $A = 1$ and $B = 0$, $Z_a = Z_c$ and $Z_b = \infty$.

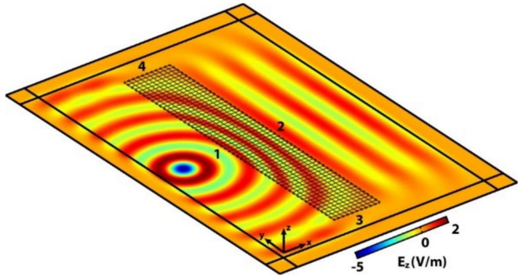
Additionally, if $\Delta = \kappa = 1$,

$$\bar{\bar{\mu}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ and } \varepsilon_z = \varepsilon_0$$

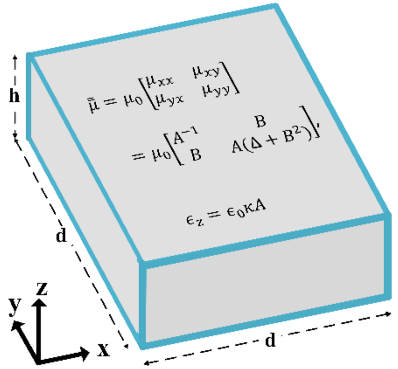
which represents free space.

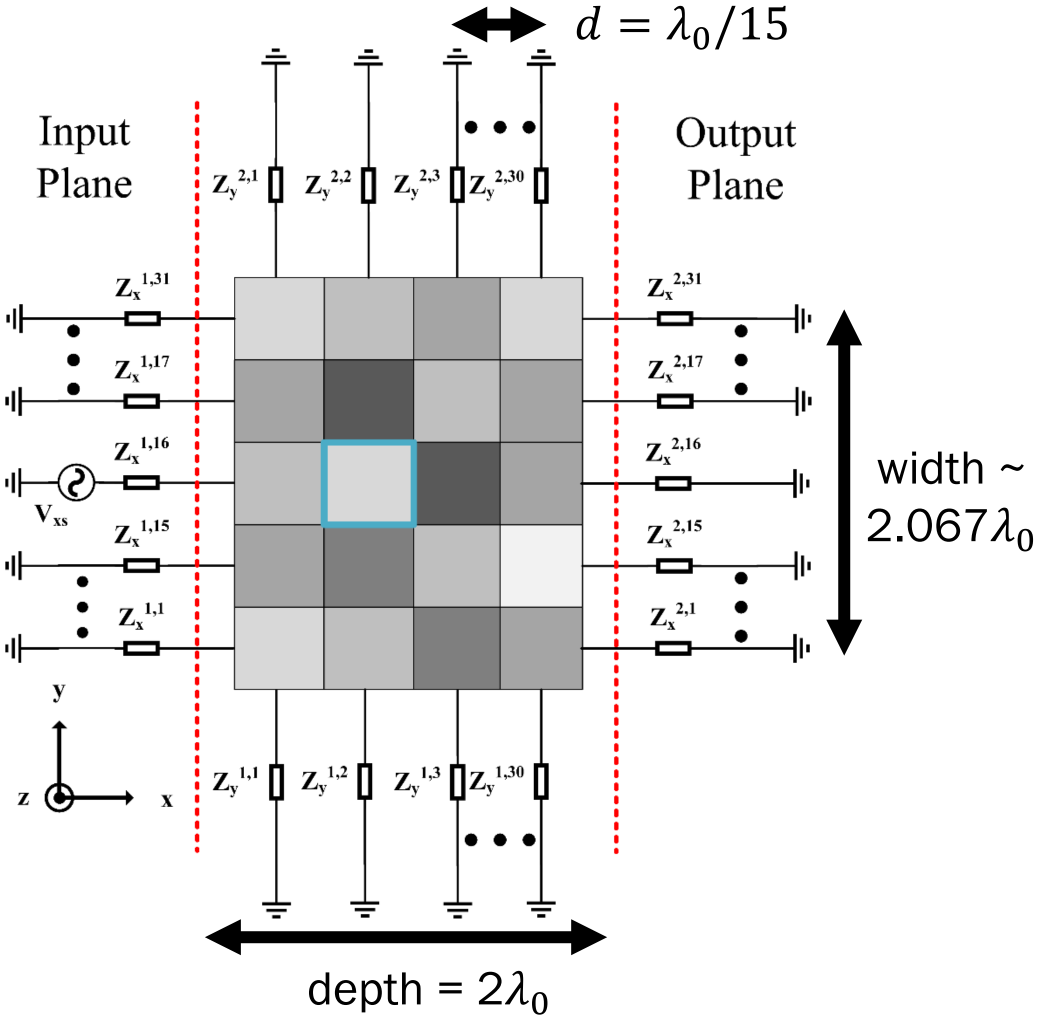
Collimator Design

- A collimator is a device that produces a parallel beam of rays as shown here



- The proposed device transforms a point source (voltage source) excitation into a collimated beam with a trapezoidal taper at 10 GHz.
- The metamaterial is surrounded by free space($\Delta = \kappa = 1$), therefore the unit cells along the perimeter are terminated with free space wave impedance = 377 Ω.
- Each unit cell has two free design variables, A and B. which are optimized within the following range
  - A: -1 → 5
  - B: -2 → 2
- Initially, $A = 1$ and $B = 0$ throughout the grid, which represents free space





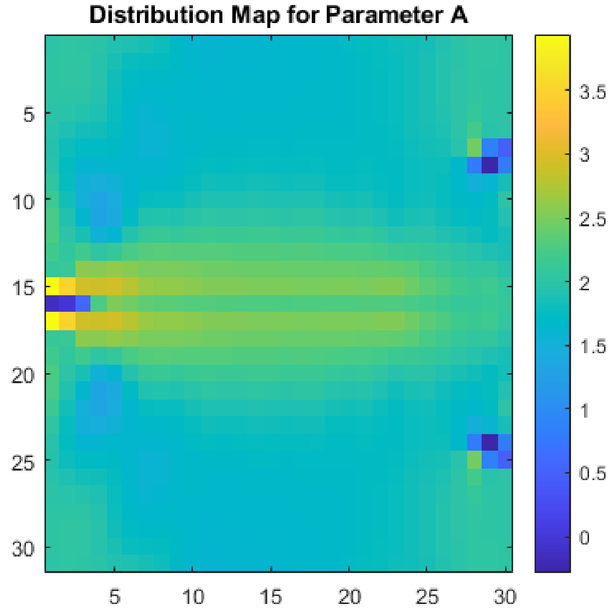
Collimator Design

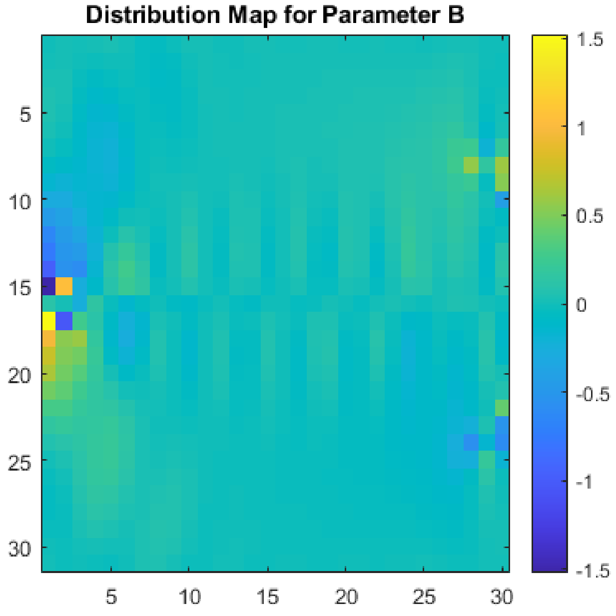
- The voltage source excites the center of the input aperture with 34.3 V with a source resistance = 147 Ω. Hence, the maximum available power from the source is 1 W.

- The cost function is imposed such that it penalizes the mismatch between the desired voltage profile and the voltage profile achieved by the device at the output aperture.

- Symmetry is imposed between the top and bottom half plane, since the desired voltage profile is symmetric. Therefore, the inverse design procedure optimizes 960 variables.

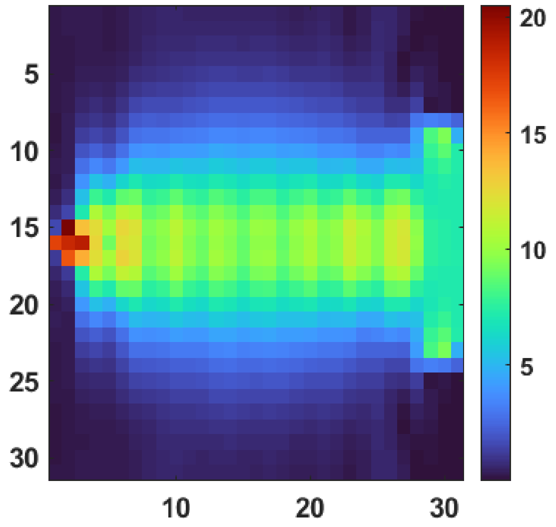
- The 2D plots of the optimized parameters over the metamaterial are shown here

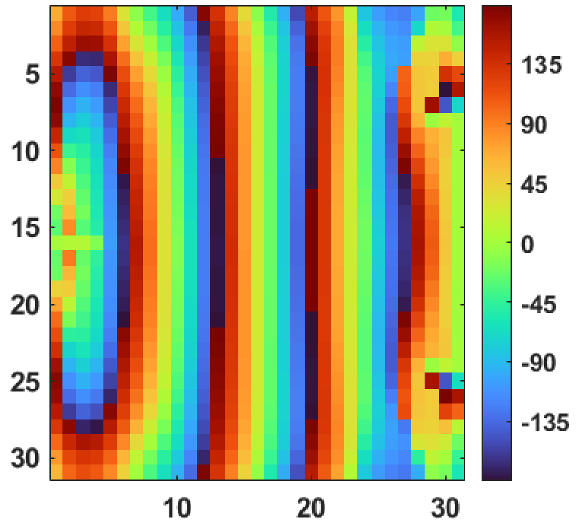


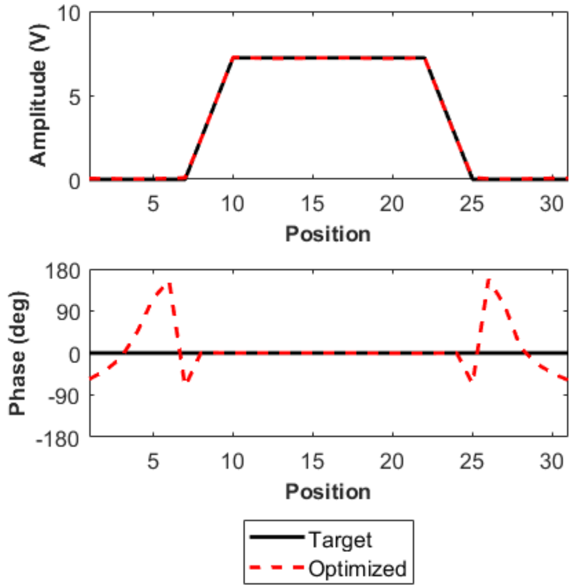


Collimator Performance

- The optimized material parameters are largely adiabatic, with localized inhomogeneous regions near the source (at the input plane) and along the output plane, where the voltage profile tapers
- The PMM initially focuses the power within the center of the grid, and subsequently spreads the power to meet the target voltage profile. The PMM gradually collimates the phase within $10d$, in the same manner as a gradient index lens.
- The amplitude and phase profile achieved is in excellent agreement with the target profile.
- The power delivered to the device is 1 W. The power at the output aperture is 69% of the power delivered.







Concluding Remarks

- All-angle reflectionless media were reviewed, and a tensor transmission-line representation of perfectly-matched media was derived
- An inverse-design procedure was then applied to design a PMM (2D electrical network) which transforms a point source excitation into a collimated beam with a prescribed amplitude taper.
- The proposed perfectly-matched metamaterials and their inverse design provide a new paradigm for the design of electromagnetic devices with complex functionalities
- These devices are devoid of inter-cell reflections and perfectly impedance-matched to the surrounding medium. Therefore, they rely on refractive effects and promise true time-delay devices with broadband operation.

Thank You

This work was supported by the National Science Foundation through the Grant Opportunities for Academic Liaison with Industry (GOALI) Program under Grant 1807940.