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WE1E-3

Time-Reversal Source Reconstruction with Arbitrary-Order Kurtosis

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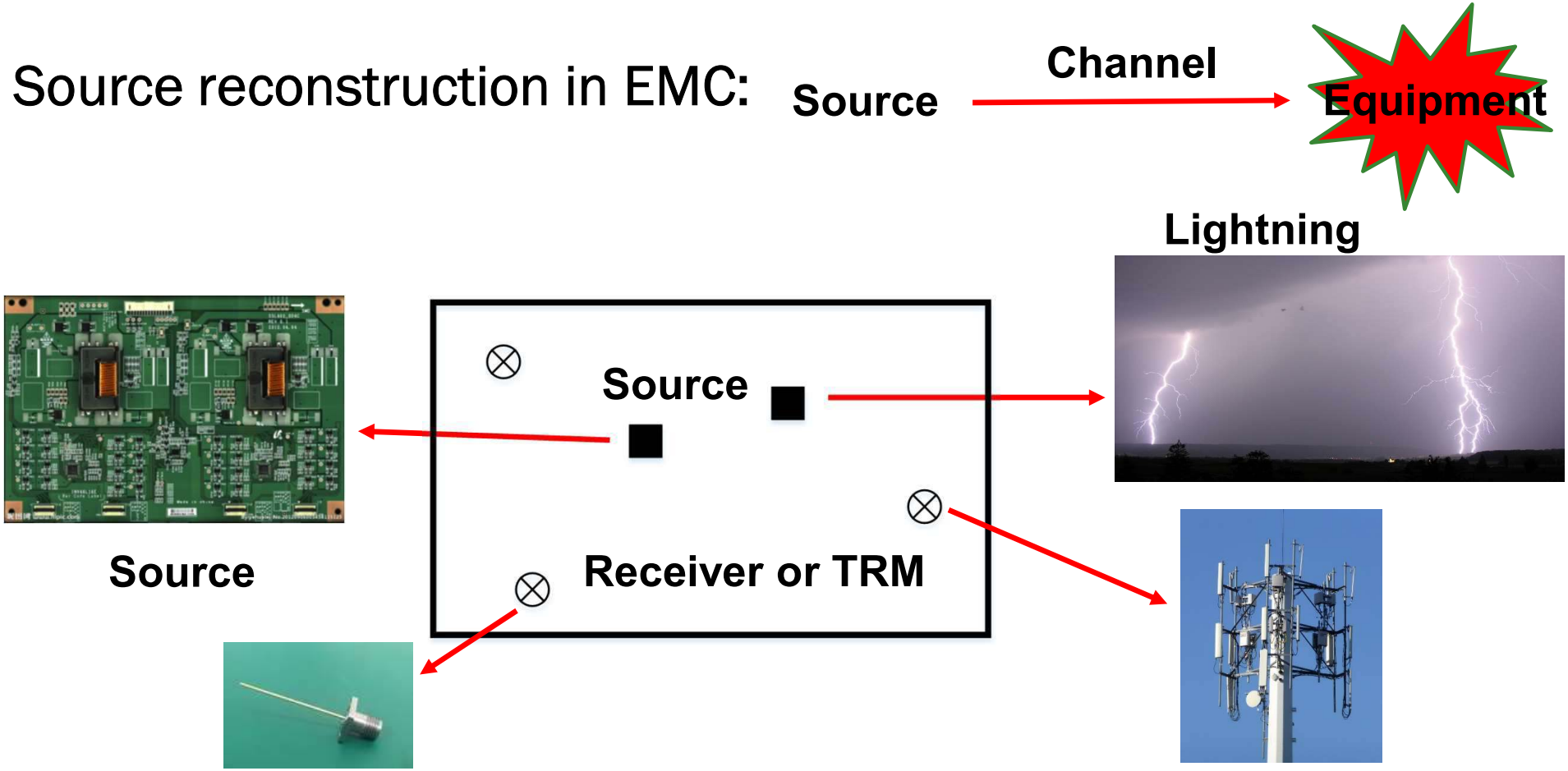
Outline

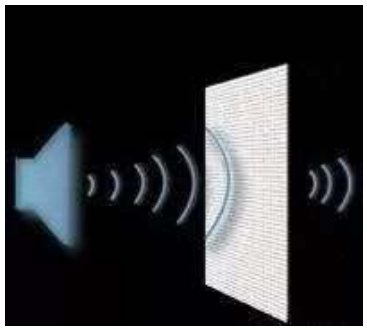
- Problem Description
- The Conventional Time-Reversal (TR) Method
- The Time Kurtosis Method
- The Band-Limited TR
- The Proposed Arbitrary-Order Kurtosis
- Summary
- Future Work

Problem Description

- Source reconstruction in EMC:



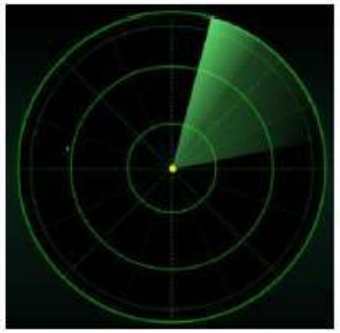
Background-TR Applications



Acoustics

Communications

Localization



Radar Imaging

EMI/EMC

Electromagnetic problems

The Conventional TR Method

Essence: phase conjugation

Cause → Effect

Forward process (Measurements)

Phase-1

Q sources

$$x_q[n] = a_q \delta[n]$$

Field propagation

$h_{pq}[n]$

Received signals at the P sensors

$$y_p[n] = \sum_{q=1}^{Q} a_q h_{pq}[n]$$

Reversed in time

$$y_p^r[n] = y_p[N-n]$$

Reversed signals are re-injected by sensors

Field propagation

$h_{qp}[n]$

Reconstructed sources

$$s_q[n] = \sum_{p=1}^{P} h_{qp}[n] \otimes y_p^r[n]$$

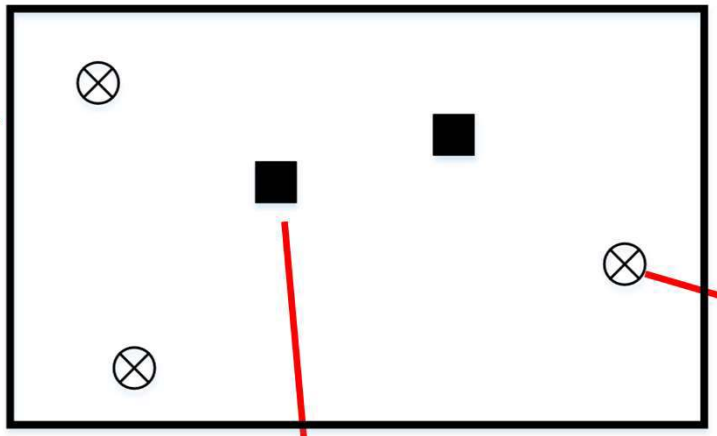
Concentration at source locations

Backward simulation (Time-reversal)

Phase-2

Effect → Cause

The Conventional TR Method



☐ A closed electromagnetic cavity

Multipath effects

☐ Sources/Antennas

☐ Sensors or Time-reversal mirror(TRM)

Measuring signals;
Inversing signals;
Simulating.

Two Unrealistic Assumptions:

1) sources start at the same time;
2) sources are impulsive

The Time Kurtosis

- The time kurtosis for direct source localization:

Time kurtosis: measure of the field temporal distribution at a spatial node – function of node position (*i*,*j*,*k*)

$$\beta_t(i,j,k) = \frac{\frac{1}{N}\sum_n \left\{E(i,j,k,n) - \overline{E}(i,j,k)\right\}^4}{\left[\frac{1}{N}\sum_n \left\{E(i,j,k,n) - \overline{E}(i,j,k)\right\}^2\right]^2}$$

Electric field at discrete node location (i,j,k) and at time step n

The electric field averaged over time n at a node location (i,j,k)

Two Unrealistic Assumptions:
1) sources start at the same time;
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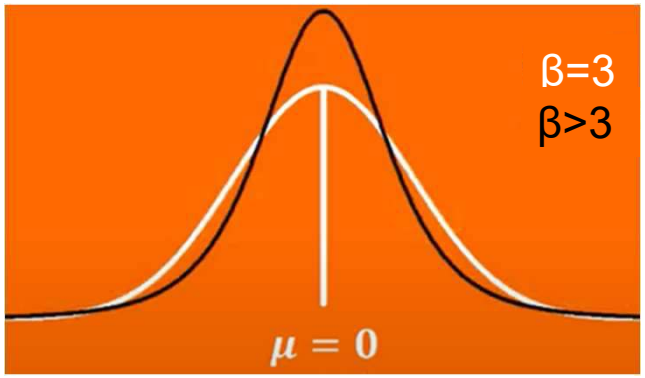
-Feng, X. Y., Chen, Z., Xu, Z. M., & Li, J. (2022). Time-reversal source reconstruction with space and time kurtoses. *IEEE Transactions on Antennas and Propagation*, 70(6), 4766-4773.

The Time Kurtosis

The two distributions have the same standard deviation



The black distribution has a larger kurtosis

For a transient source, the function of signal should has a high kurtosis value at a specific location
→ identification of the location of the source excitation

→

During the second phase of the TR, we compute the kurtosis at each time step, plot the kurtosis versus spatial nodes
→ locations of the largest kurtosis values are the source excitation positions.

Two Unrealistic Assumptions: **1) sources start at the same time;**
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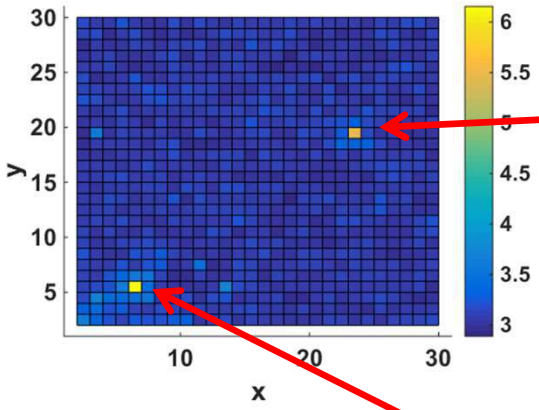
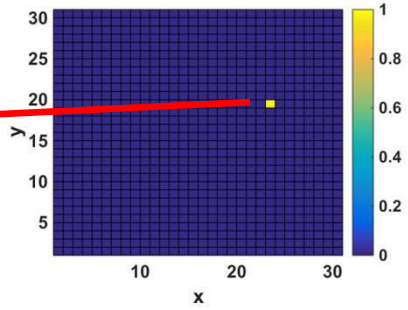
The Time Kurtosis



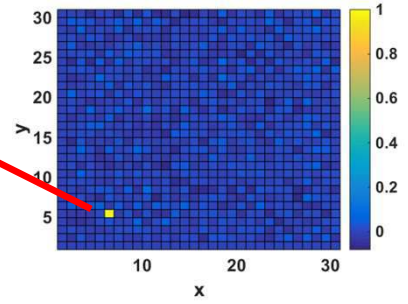
Time kurtosis

$$\beta_t(i,j,k)=\frac{\frac{1}{N}\sum_n\left\{E(i,j,k,n)-\overline{E}(i,j,k)\right\}^4}{\left[\frac{1}{N}\sum_n\left\{E(i,j,k,n)-\overline{E}(i,j,k)\right\}^2\right]^2}$$



Starts at n=800

Original sources



Starts at n=2000

Two Unrealistic Assumptions: **1) sources start at the same time;** **2) sources are impulsive**

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The Band-Limited TR

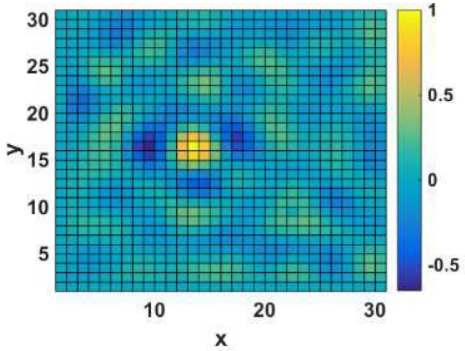


Fig. 1 TR source reconstruction with 10-20 GHz signal

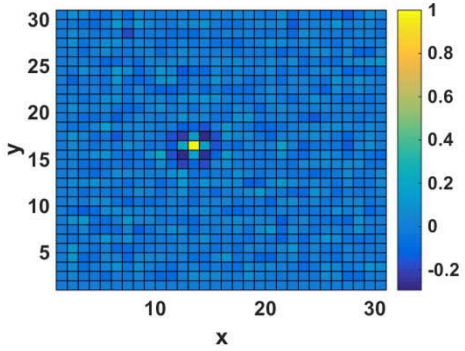


Fig. 2 TR source reconstruction with 10-40 GHz signal

Algorithm #1:

$$y^c[n] = \sum_{k_f=k_L}^{k_H} \left|Y^c[k_f]\right| \cos\left(2\pi n \frac{k_f}{N} + \angle Y^c[k_f]\right)$$

Algorithm #2:

$$y^c[n] = \sum_{k_f=k_L}^{k_H} \left|Y^c[k_f]\right| \sin\left(2\pi n \frac{k_f}{N} + \angle Y^c[k_f]\right)$$

Two Unrealistic Assumptions:

1) sources start at the same time;

2) sources are impulsive

-Feng, X. Y., Chen, Z., Li, J., Cai, J., & Liang, J. C. (2022). Electromagnetic time kurtosis for time-reversal source reconstruction with band-limited signals. In *2022 IEEE/MTT-S International Microwave Symposium (IMS)*.

The Proposed Arbitrary-Order Kurtosis

- The definition of arbitrary-order kurtosis:

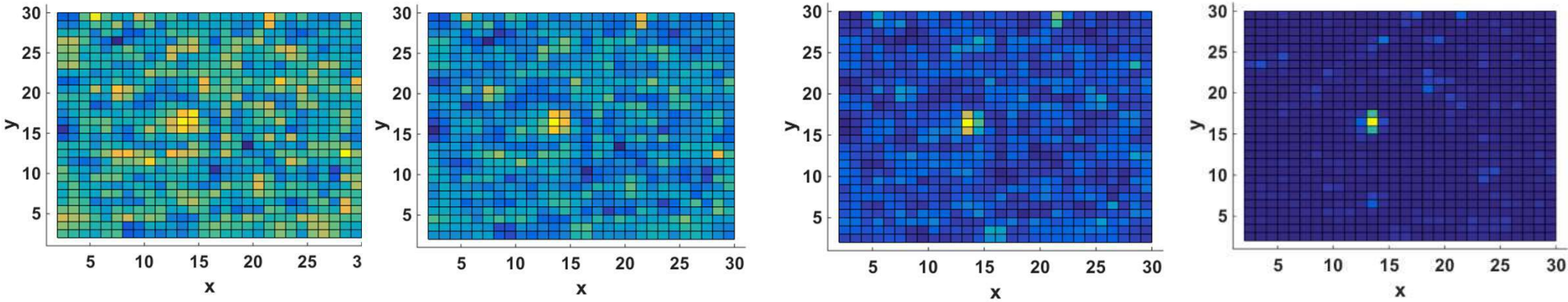
$$\frac{\frac{1}{N}\sum_{n}\left\{E(i,j,k,n)-\overline{E}(i,j,k)\right\}^{2\alpha}}{\left[\frac{1}{N}\sum_{n}\left\{E(i,j,k,n)-\overline{E}(i,j,k)\right\}^{2}\right]^{\alpha}}$$

Order α

Two Unrealistic Assumptions: **1) sources start at the same time;**
2) sources are impulsive

The Proposed Arbitrary-Order Kurtosis

α = 2 (time kurtosis) α = 3 (3rd order kurtosis) α = 5 (5th order kurtosis) α = 9 (9th order kurtosis)



(a) (b) (c) (d)

Fig. 3 TR source reconstruction with high-order kurtosis. (the orders are chosen arbitrarily)

Summary

- Band-limited TR has limited resolutions.
- Higher order kurtosis can be applied to help reconstruct source effectively in realistic applications with band-limited TR.
- In our examples, when the order $\alpha > 2$, the spatial distribution of higher-order kurtosis can make the source regions more distinct in the TR source reconstruction with much lower error.

Future Work

- Future research of the time-reversal method may include the following topics but not limited to:
  - time-reversal bandwidth and spatial resolution;
  - experimental hardware validations of the kurtosis method;
  - source reconstruction in frequency domain;
  - use in millimeter-wave and THz frequency ranges.

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