Near and Far Field Characteristics of Two in Line Graphene Coated Dielectric Nanowires Excited by Modulated Electron Beam

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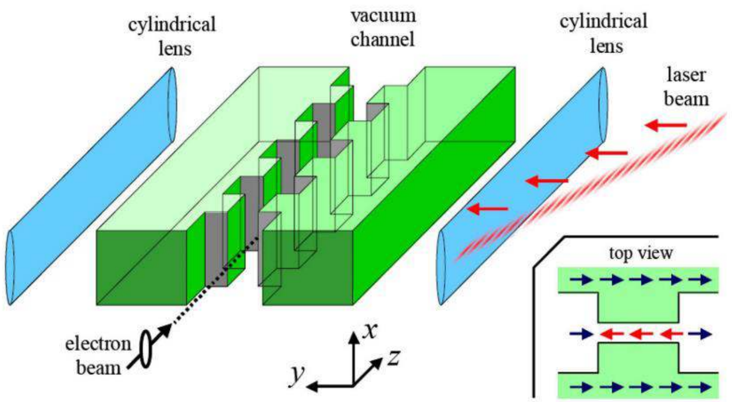
CONTENTS

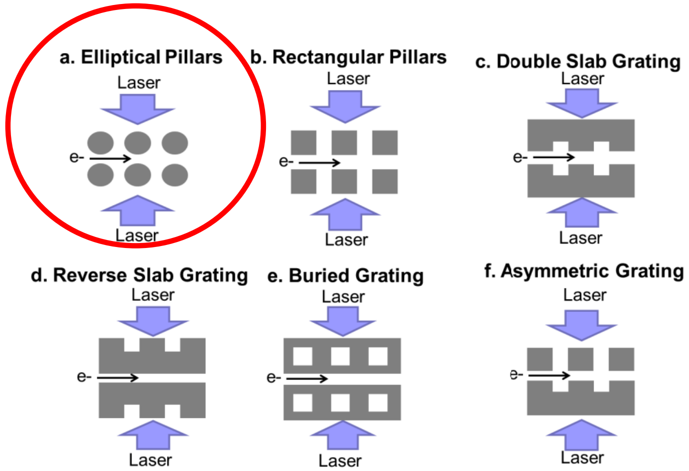
- Goal and motivation
- Problem formulation
- Basic equations
- Numerical results
  - Total scattering and absorption cross section spectra
  - In-resonance supermode near and far field patterns
- Conclusions

GOAL AND MOTIVATION

The dielectric laser accelerators (DLA) are micrometer-scale structures made of silicon or other dielectric materials and illuminated by external laser light. They eliminate metal components and therefore overcome electrical breakdown limitations of conventional particle accelerators in the presence of high electric fields.

The principal elements of DLAs are diffraction gratings.





Wootton, Kent & Mcneur, Josh & Leedle, Kenneth. (2016). Dielectric Laser Accelerators: Designs, Experiments, and Applications. Reviews of Accelerator Science and Technology.

Our goal is accurate quantification of the resonance effects in DR from two circular nanowires covered by graphene and placed in line with the beam trajectory that is close to the DLA-related applications.

PROBLEM FORMULATION

We assume two circular graphene-covered dielectric rods with radius *a* and dielectric permittivity *ε* at the distance *L* from each other.

Time dependence is $\boxed{e^{-i\omega t}}$

We introduce the global Cartesian and polar coordinates, $\vec{r} = (x, y) = (r, \varphi)$ $\quad x = r\cos\varphi,\; y = r\sin\varphi$

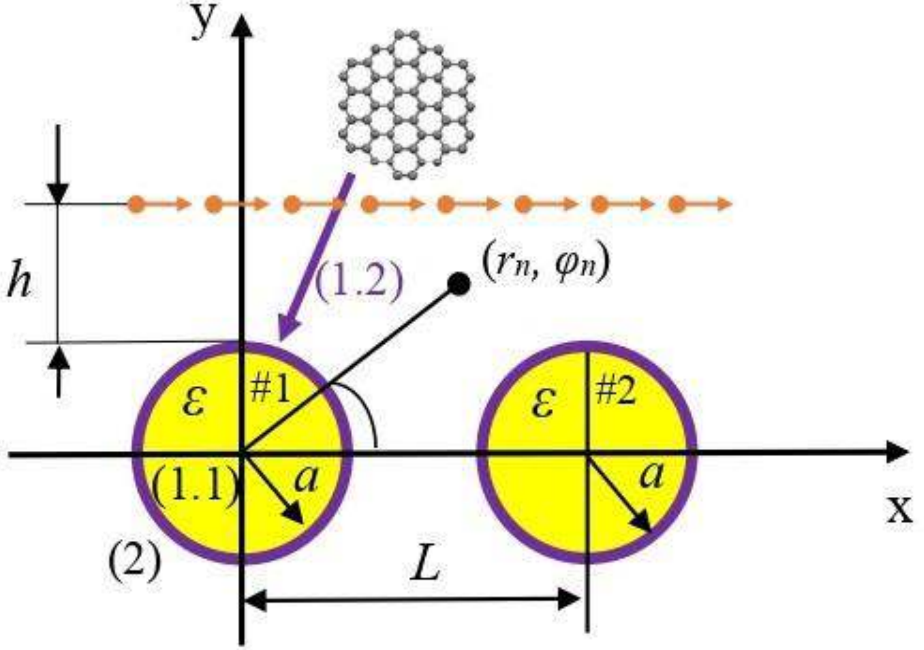
The incident wave is the field of a sheet current moving along the straight trajectory with velocity $v = \beta c \;\; (\beta < 1)$

The beam charge density is $\rho = \rho_0 \delta(y-h)\exp[i(kx/\beta - \omega t)]$ (1)

The incident field is the H-polarized slow surface wave propagating along the beam trajectory

$$H_z^0(x, y) = A\beta \operatorname{sign}(y-h) e^{-q|y-h|} e^{i(k/\beta)x} \quad (2)$$

where $q = k\gamma/\beta$, $\gamma = (1-\beta^2)^{1/2}$



a = radius
ε = dielectric permittivity
L= distance between wires' centers
h = impact parameter

GRAPHENE'S CONDUCTIVITY DESCRIPTION

The most widely adopted today quantum model of the electron mobility in the graphene monolayer is the Kubo model. Here, the graphene thickness is considered zero, and its surface conductivity is $\sigma(\omega,\mu_c,\tau,\mathrm{T})$. This value consists of two contributions, $\sigma=\sigma_{\mathrm{intra}}+\sigma_{\mathrm{inter}}$, which are intraband and interband conductivities

$$\sigma_{\mathrm{intra}}=\frac{\Omega}{(1/\tau-\mathrm{i}\omega)}\left\{\frac{\mu_c}{\mathrm{k_B T}}+2\ln\left[1+\exp\left(-\frac{\mu_c}{\mathrm{k_B T}}\right)\right]\right\},\qquad \Omega=\frac{\mathrm{q_e^2 k_B T}}{\pi\hbar^2},\qquad \sigma_{\mathrm{inter}}=\frac{\mathrm{i}\mathrm{q_e^2}}{4\pi\hbar}\ln\frac{2|\mu_c|-(\omega+\mathrm{i}\tau^{-1})\hbar}{2|\mu_c|+(\omega+\mathrm{i}\tau^{-1})\hbar}.$$

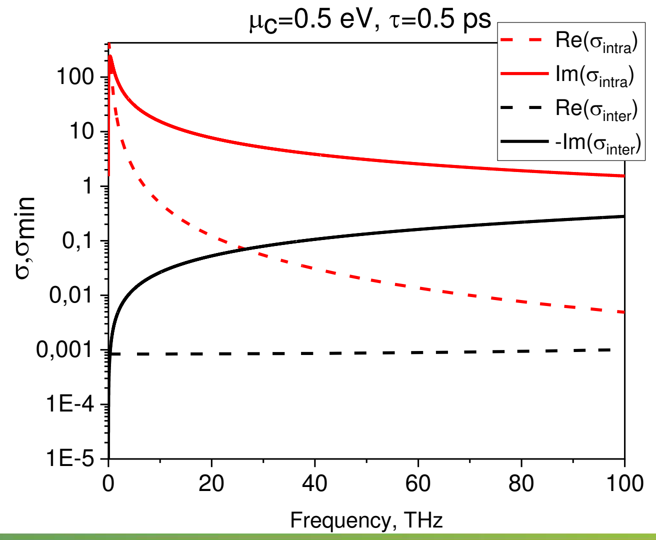
Then, the normalized (i.e. dimensionless) surface impedance of graphene is $\boxed{\mathrm{Z}(\omega)=\mathrm{Z}_0^{-1}\left(\sigma_{\mathrm{intra}}+\sigma_{\mathrm{inter}}\right)^{-1}}$

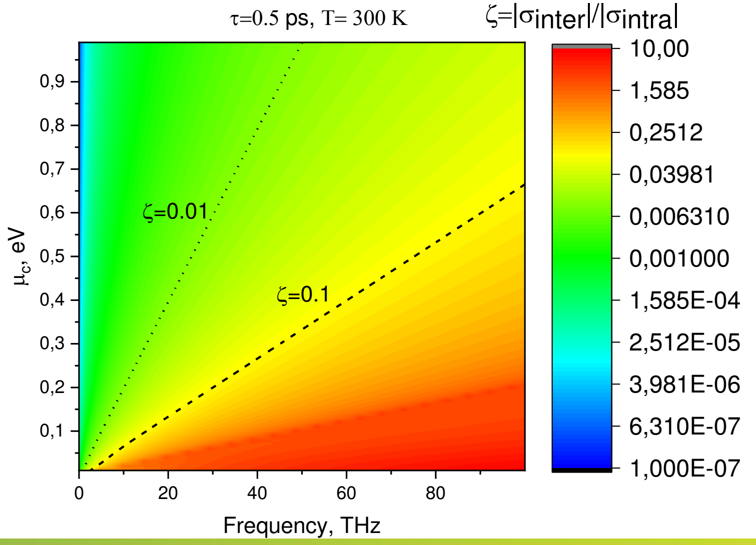
The plasmon frequencies of single nanowire are given by

$$\boxed{\mathrm{f_m^P}\approx\frac{1}{2\pi}\left[\frac{\mathrm{mc}\Omega}{\mathrm{a}(\varepsilon+1)}\right]^{1/2}}$$

Additionally, when $\mu_c \gg \mathrm{k_B T}$ then Ω is proportional to potential μ_c.

The relative contribution of two terms into *Z*





BASIC EQUATIONS

We search for the magnetic field written as follows:

$$H^{tot} = \begin{cases} H^{int(p)}, & r_p < a_p, \quad p = 1,2 \\ H^0 + H^{ext}, & r : \{r_p > a_p,\ p = 1,2\} \end{cases} \qquad (3)$$

The field in partial domains can be expressed as

$$H^{int(p)}(r,\varphi) = \sum_{n=-\infty}^{\infty} y_n^{(p)} J_n(k\alpha r_p) e^{in\varphi_p}, \ r_p < a, \ \ p = 1,2 \qquad (4)$$

$$H^{ext}(r,\varphi) = \sum_{p=1,2} \sum_{n=-\infty}^{\infty} z_n^{(p)} H_n^{(1)}(kr_p) e^{in\varphi_p}, \ r_p > a \qquad (5)$$

where $y_n^{(p)}, z_n^{(p)}$ **are unknown coefficients,** $H_m(\cdot)$ **and** $J_m(\cdot)$ **are the first-kind Hankel and the Bessel functions, respectively.**

The resistive boundary conditions at the rod contours

$$r_p = a, \ 0 \le \varphi_p < 2\pi,$$

$$E_{\varphi_p}^{int(p)} = E_{\varphi_p}^{0} + E_{\varphi_p}^{ext}, \qquad (6)$$

$$\boxed{E_{\varphi_p}^{int(p)} + E_{\varphi_p}^{0} + E_{\varphi_p}^{ext} = 2ZZ_0\left[H^{int(p)} - H^0 - H^{ext}\right]} \qquad (7)$$

where $p = 1,2$, $Z_0 = \sqrt{\mu_0/\varepsilon_0}$ **is the free space impedance,** Z **is surface impedance of graphene**

Substituting into (6) and (7) the series (4) and (5) and a similar series for the beam field (2), using Graf's theorem, and introducing new scaled unknowns $x_n^{(p)} = z_n^{(p)} w_n$ **where** $w_{n<0} = (-1)^n w_{n>0}, \quad w_{n>0} = n!(2/ka)^n,$

we derive two coupled Fredholm 2-nd kind matrix equations (*p* ≠ *j* =1,2),

$$\boxed{x_m^{(p)} \pm V_m D_m^{-1} \sum_{n=-\infty}^{+\infty} w_n H_{m-n}(kL) x_n^{(j)} = F_m^{(p)} D_m^{-1},} \qquad (8)$$

where $V_m = J'_m - iZ\left[J'_m \frac{\alpha J_m(k\alpha a)}{J'_m(k\alpha a)} - J_m\right]$

$$D_m = w_m \left\{ H'_m - iZ\left[H'_m \frac{\alpha J_m(k\alpha a)}{J'_m(k\alpha a)} - H_m\right]\right\}$$

$$F_m^{(p)} = iZ\left[g_m'^{(p)} \frac{\alpha J_m(k\alpha a)}{J'_m(k\alpha a)} - g_m^{(p)}\right] - g_m'^{(p)}$$

$$g_m^{(1,2)} = -Ae^{-q(s+a+h)} i^m J_m (1-\gamma)^m \beta^{-m+1}$$

and the omitted arguments of the cylindrical functions are ka **.**

Fredholm equation guarantees convergence of the numerical code.

SCATTERING AND ABSORPTION CROSS- SECTIONS

At $r \to \infty$, we express the scattered field as

$$H^{sc}(r,\varphi) = (2/i\pi kr)^{1/2}\Phi(\varphi)\exp(ikr)$$

where the angular scattering pattern is a function of $z_m^{(1,2)}$

$$\Phi(\varphi) = \sum_{m=-\infty}^{+\infty} (-i)^m J_m \left[e^{-\frac{1}{2}ikL\sin\varphi} z_m^{(1)} + e^{\frac{1}{2}ikL\sin\varphi} z_m^{(2)} \right] e^{im\varphi},$$

Partial **scattering cross-sections** (SCS) are

$$\boxed{\sigma_{sc}^{(1,2)} = \frac{2}{\pi k A^2} \int_0^{\pm\pi} |\Phi(\varphi)|^2 d\varphi}$$

Although the dielectric nanorods can be assumed lossless, the graphene covers are sizably lossy. Then, the partial **absorption cross sections** (ACS) are

$$\boxed{\sigma_{abs}^{(1,2)} = \pi a \frac{\operatorname{Re} Z}{A^2 |Z|^2} \sum_{n=-\infty}^{\infty} \left| A_n^{(1,2)} \right|^2}$$

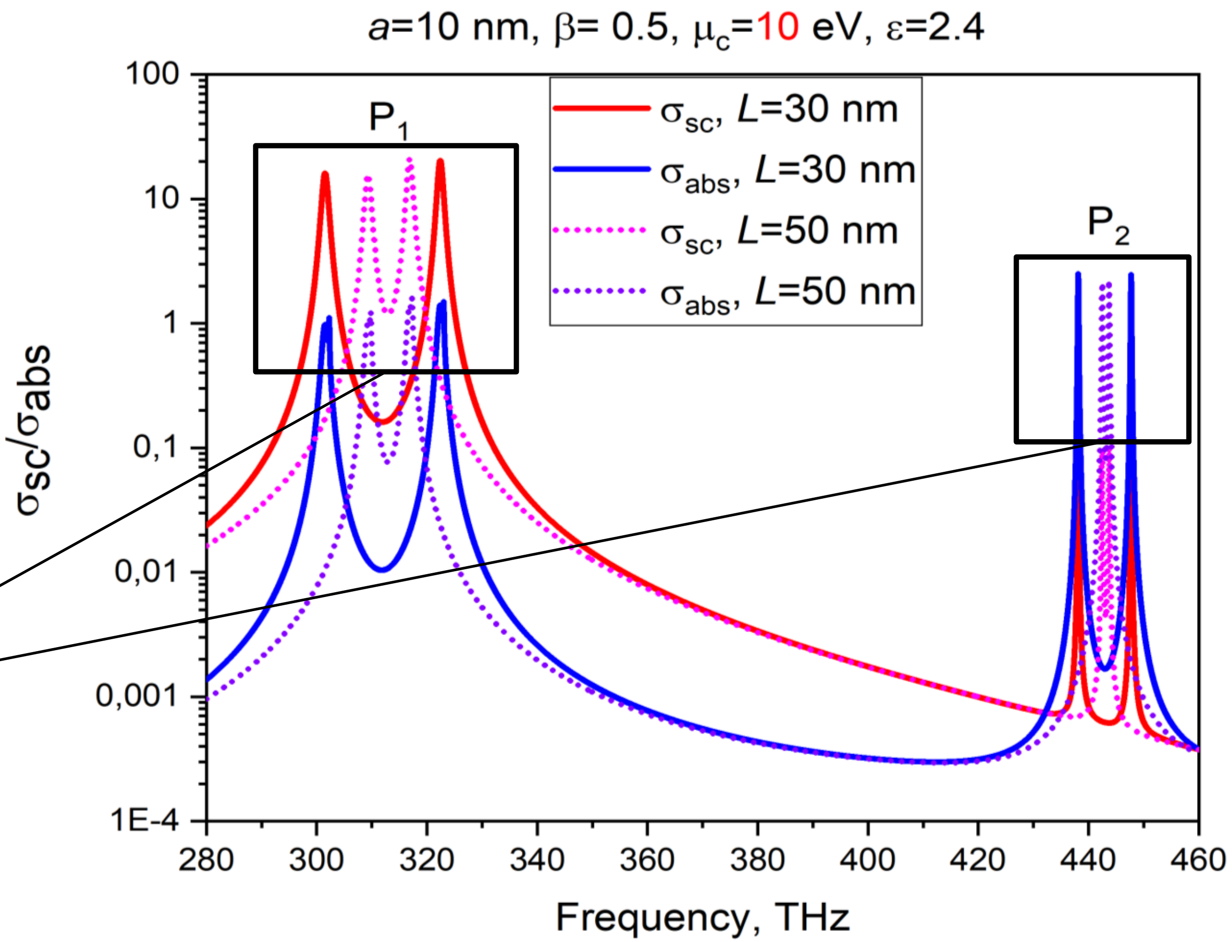
where $A_n^{(1,2)} = g_n'^{(1,2)} + z_n^{(1,2)} H_n' + J_n' \sum_{m=-\infty}^{\infty} (\pm i)^{m-n} z_m^{(2,1)} H_{n-m}(kL)$

The sum of all partial SCS and ACS is the extinction cross-section, σ_{ext}

This is **the Optical Theorem** adapted to the DR effect of a modulated beam of electrons.

NUMERICAL RESULTS

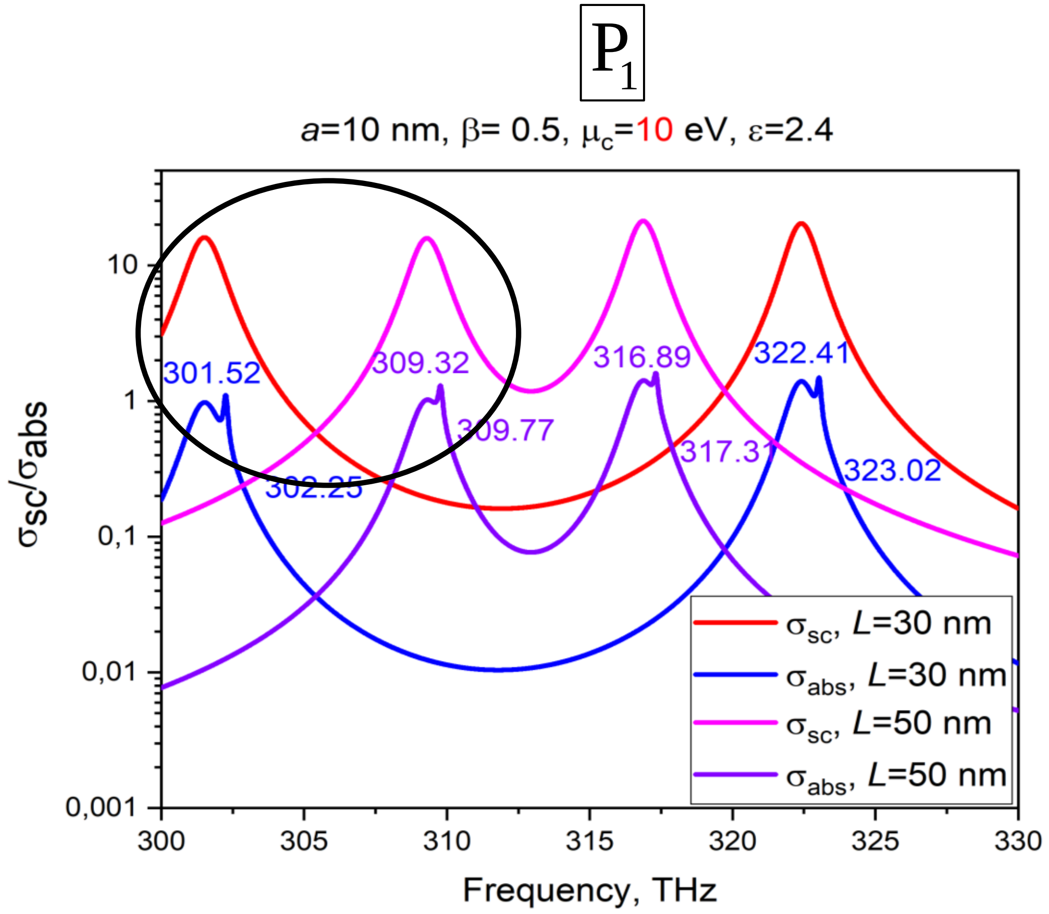
The Total Scattering and Absorption Cross Sections spectra in the infrared range (from 280 nm to 460 nm) for two distances between the nanorods
L = 30 nm and L = 50 nm are presented.



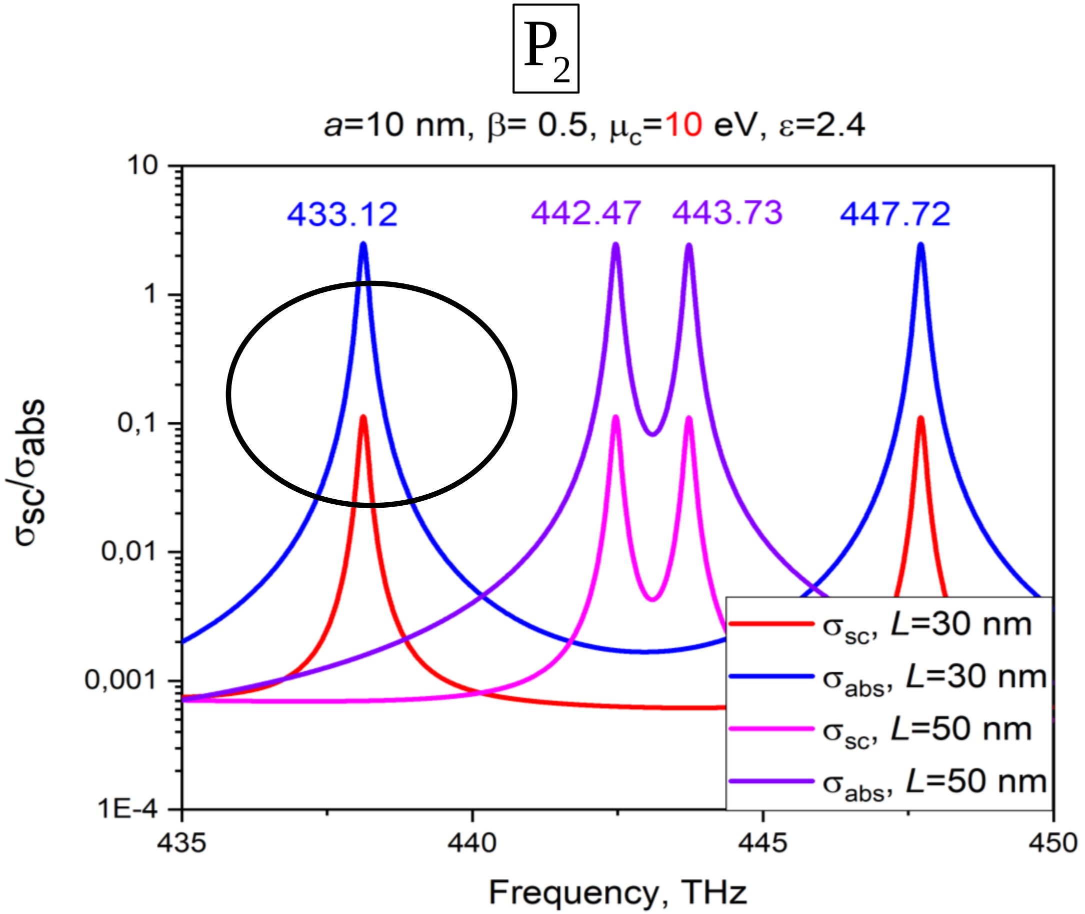
The dipole supermode *P*1 and the quadrupole supermode *P*2 quartets are shown closer on the next slide

NUMERICAL RESULTS

Zooms around the frequencies of the supermodes *P*1 and *P*2



The quartets are visible

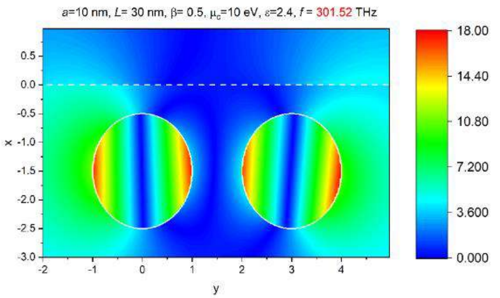


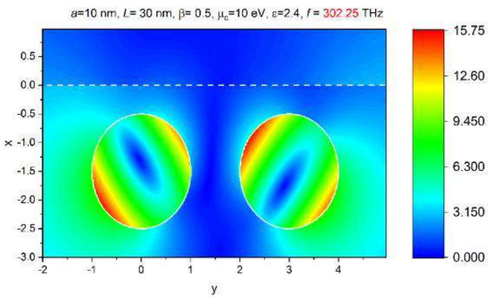
The quartets are not resolved

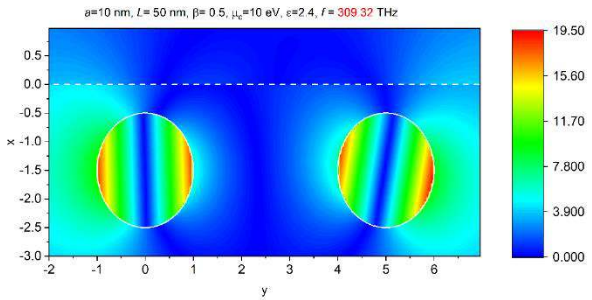
NUMERICAL RESULTS

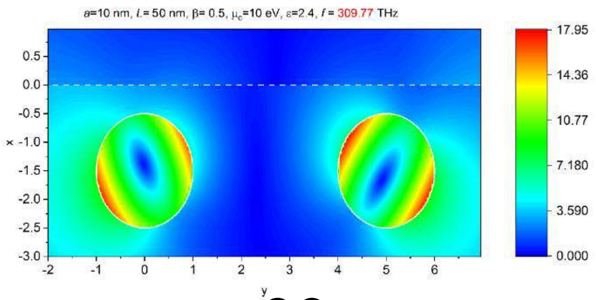
Near magnetic and far field patterns of supermode P_1 .

(a) **L = 30 nm** (b) (c) **L = 50 nm** (d)

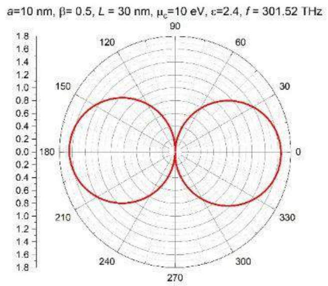


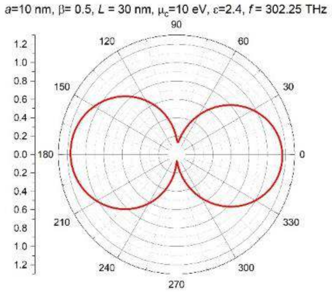


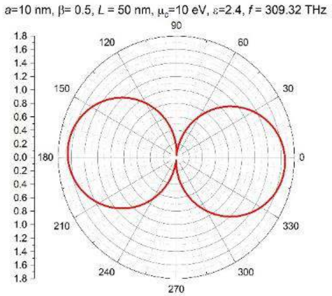


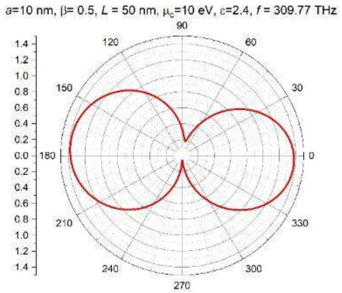


OE **OO** **OE** **OO**









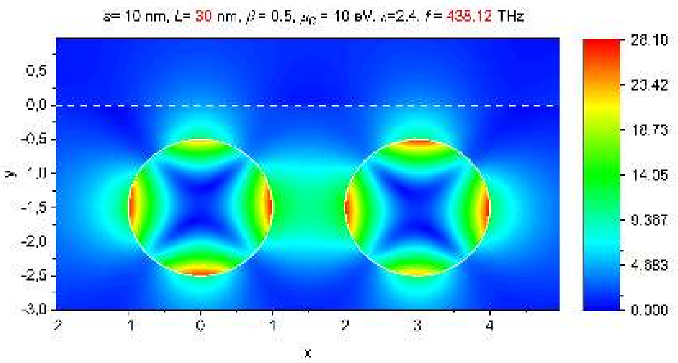
f = 301,52 THz f = 302,25 THz f = 309,32 THz f = 309,77 THz

NUMERICAL RESULTS

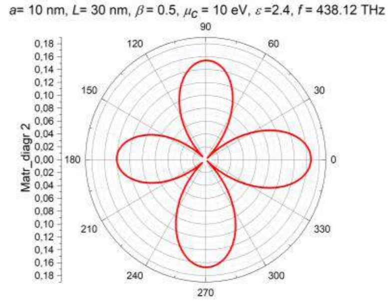
Near magnetic and far field patterns of supermode *P*2.

L = 30 nm

(a)

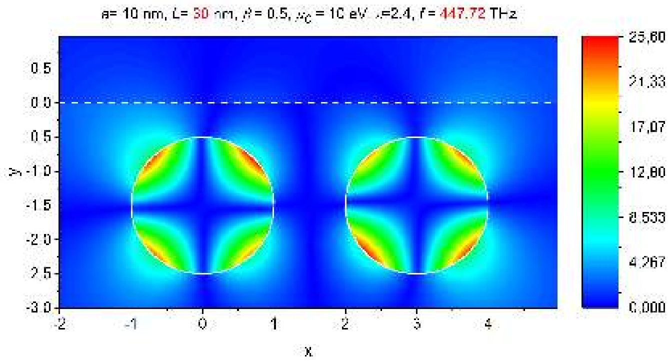


EE

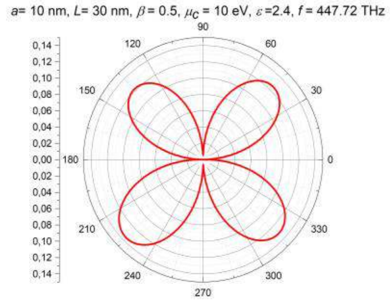


f = 433.12 THz

(b)



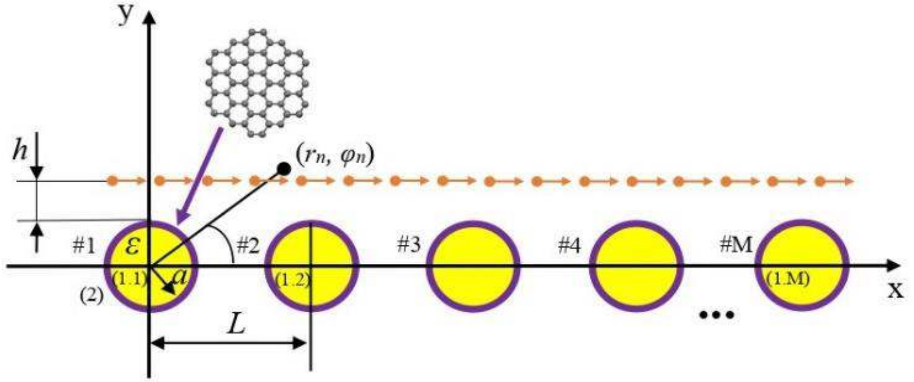
OO

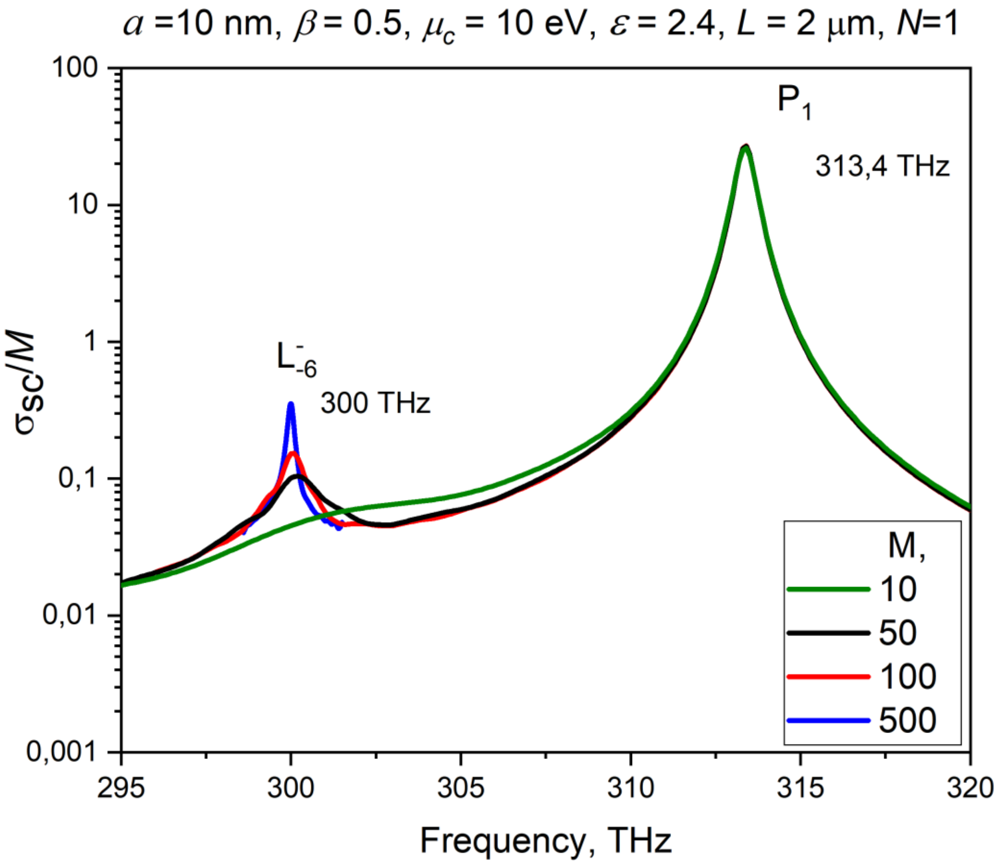


f = 447,72 THz

FURTHER RESEARCH: FINITE GRATING

Preliminary results for DR from finite grating





P_1 – plasmon modes

Note that the magnitude of this peak, if normalized by M, does not depend on the number of wires (all the curves overlap completely).

$L_{(-6)}$ - lattice resonance.

Due to analytical regularization and Fredholm 2nd kind nature of the final equations, the accuracy is far beyond any commercial code

CONCLUSIONS

We have presented basic equations and sample numerical results for the diffraction radiation from two in-line dielectric circular nanorods with graphene covers exited by the modulated electron beam.

The resonances on **the plasmon supermodes of different symmetries** have been found and discussed. By means of changing the size of the distance *L*, one can manipulate the resonance frequencies. The larger the *L*, the closer the frequencies of all four supermode resonances to the limiting value, which is the frequency of the corresponding plasmon mode of the single circular coated nanorod.

This analysis can be useful in the design of DLA sections made of low-index dielectrics, however, covered with graphene.

ACKNOWLEDGMENT

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Thank you for your interest and support

Slava Ukraine!

War is not over!

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