



We2G-5

A High-efficiency and High-accuracy Distance Measurement Technique Based on Phase Differentiation and Accumulation with FMCW radars

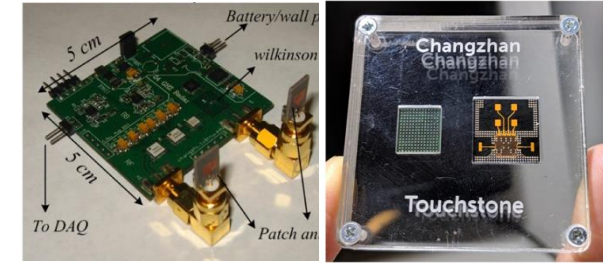
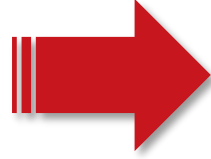
Jingtao Liu, Zesheng Zhang, Jingyun Lu, Yuchen Li, Changzhan Gu, Jun-Fa Mao

State Key Laboratory of Radio Frequency Heterogeneous Integration, and MoE Key Lab of Artificial Intelligence, Shanghai Jiao Tong University, China, 200240

Email: changzhan@sjtu.edu.cn

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Introduction



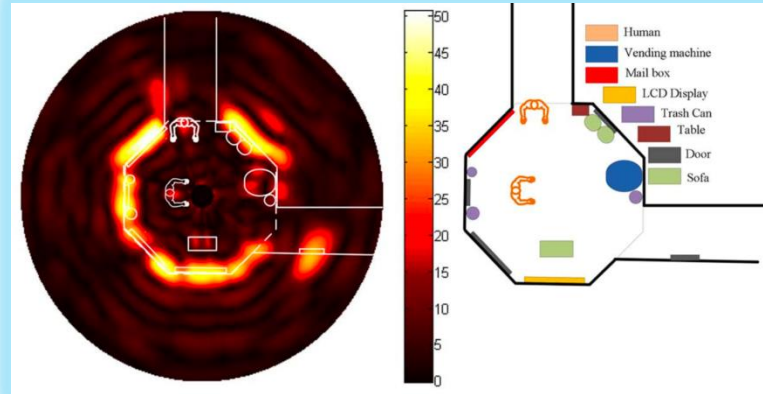
Long range ($>1\text{km}$)

Short range ($<5\text{m}$)

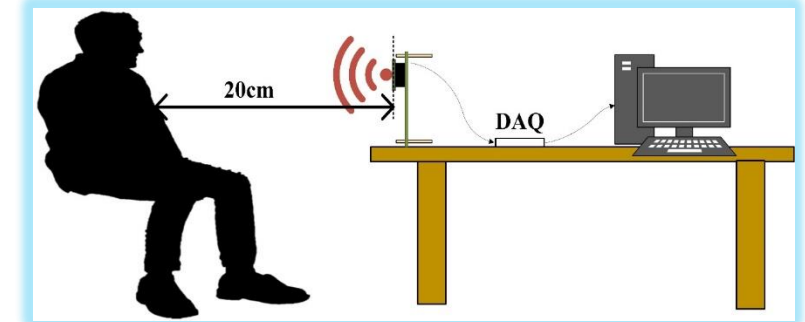
Miniaturization



Touchless human computer interaction



in-door positioning and navigation



Non-contact vital sign detection

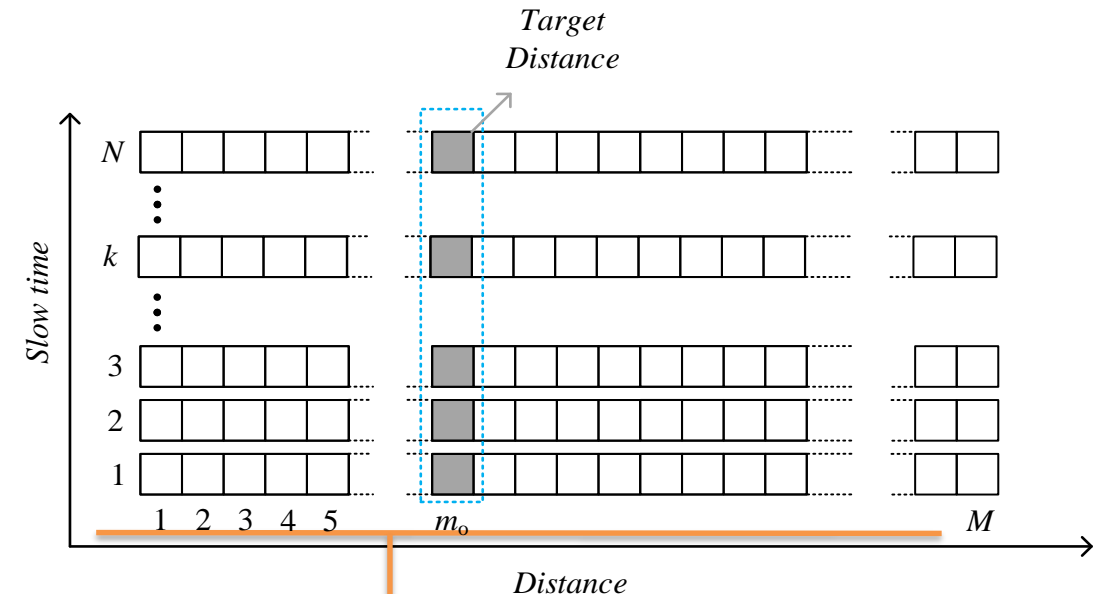
In a common FMCW radar, the IF signal, also known as the beat signal, can be modeled as follows:

$$s_b(t) = A \exp\left(j\left(\frac{4\pi BR}{T_c} \cdot t + \frac{4\pi f_c R}{c}\right)\right), t \in \left[\frac{T}{2}, \frac{T}{2}\right], \quad (1)$$

Notice two important components in (1):

- 1) The frequency component also known as the beat frequency $f_b = 2BR/T_c$
- 2) The phase item $\varphi = 4\pi f_c R/c$.

The target distance R can be derived from the beat frequency, which is usually estimated with FFT.



Limited frequency resolution → limited accuracy

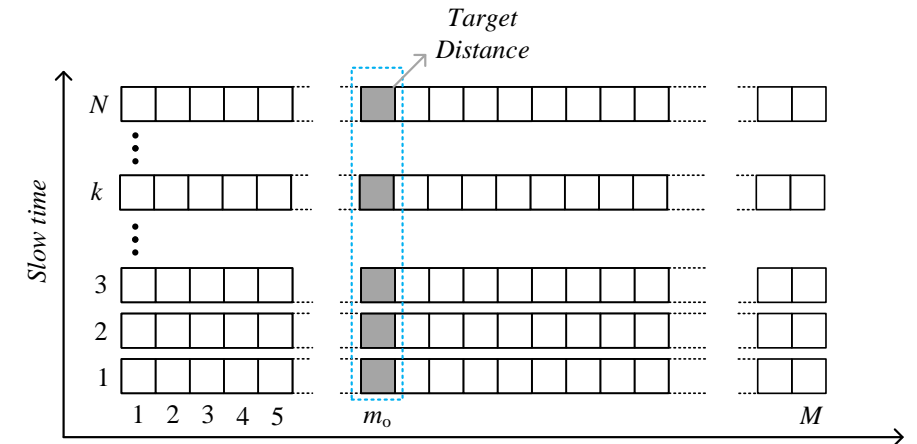
Zero-padded FFT → Large computer resource consumption → **Not suitable for Embedded device**

different interpolation methods [1-2]

- Interpolation between the discrete points
- Introduce new error / the frequency resolution is still limited

chirp z-transform (CZT)-based technique [3]

- Further CZT in the concerned frequency band
- the frequency resolution is still limited and the efficiency is still low when high resolution required



This work

- Phase Differentiation and Accumulation (PDA) within one chirp
- Frequency resolution free/ high efficiency of $O(N\log(N/3))$

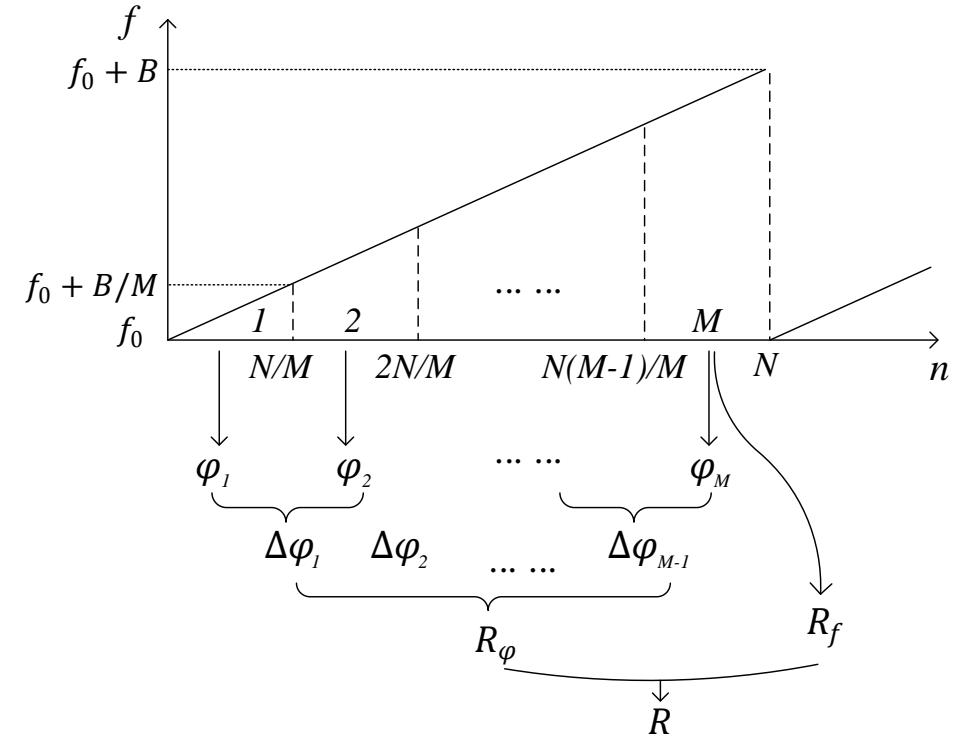
As discussed previously, the IF signal of a common FMCW radar is follows:

$$s_b(t) = A \exp\left(j\left(\frac{4\pi BR}{Tc} \cdot t + \frac{4\pi f_c R}{c}\right)\right), t \in \left[\frac{T}{2}, \frac{T}{2}\right], \quad (1)$$

The distance is usually estimated through the frequency components $f_b = 2BR/Tc$.

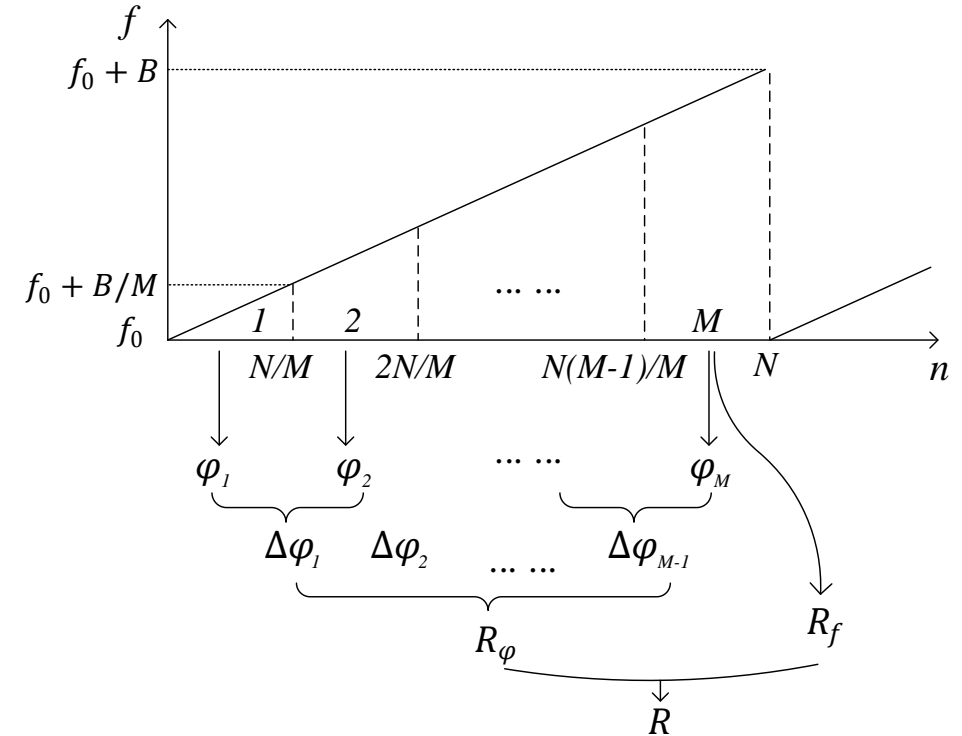
However, estimation of f_b suffers from limited frequency resolution. A high frequency resolution leads to large computer resource consumption, making it unsuitable for embedded device.

On the other hand, there is another phase components $\varphi = 4\pi f_c R/c$ in (1). Estimation of φ is frequency resolution free but it suffers from phase ambiguity.



This work combines the frequency and phase components and thus realizes a high efficient and frequency resolution free distance measurement technique. It is detailed as follows:

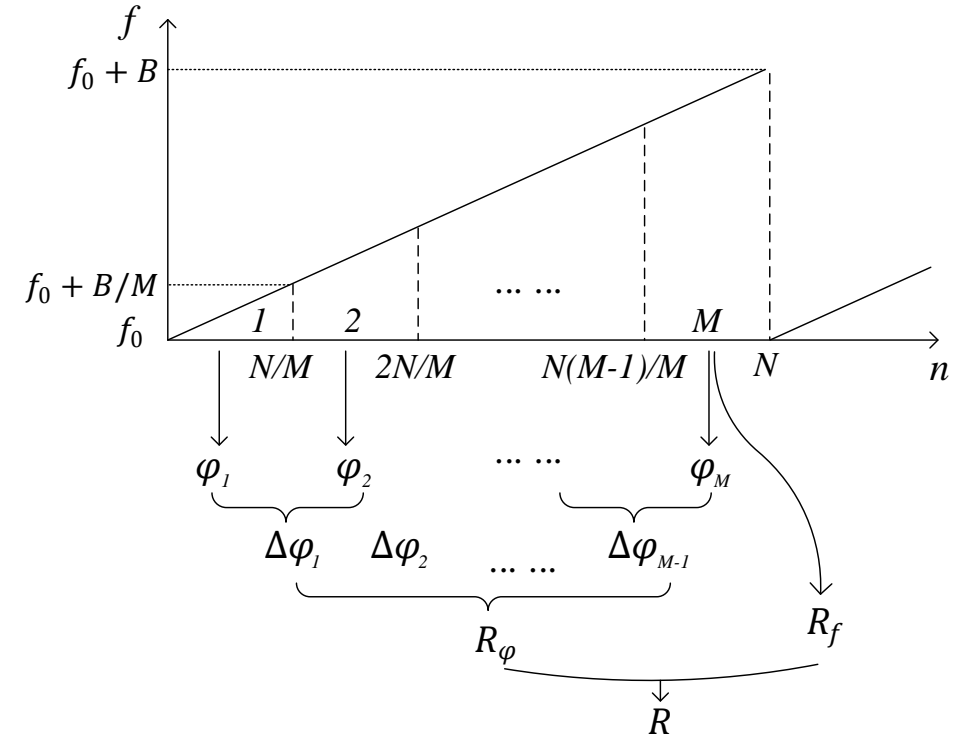
- 1) The sampled beat signal of one chirp has a length of N . Divide the chirp into M segments. Each segment can be seen as a sub-chirp with bandwidth being $\Delta f = B/M$ and center frequency being $f_{cm} = f_0 + Bm/M$, $m=1 \sim M$. Thus, its distance resolution is $\Delta R = \frac{c}{2\Delta f}$
- 2) Perform non-zero-padded FFT on each segment to get the spectra $F_m[k]$, $k=1 \sim N/M$, $m=1 \sim M$. The distance between two adjacent points of $F_m[k]$ is ΔR . Find the target peak's location of k_b . The target distance can be coarsely estimated to be $R_f = \Delta R \cdot k_b$.



3) Extract the phase of $F_m[k_b]$ to get $\varphi_m = \arg(F_m[k_b])$, $m=1 \sim M$. The adjacent phase difference is ($m=1 \sim M-1$):

$$\Delta\varphi_m = \begin{cases} \varphi_{m+1} - \varphi_m, & \varphi_{m+1} > \varphi_m \\ \varphi_{m+1} + 2\pi - \varphi_m, & \varphi_{m+1} < \varphi_m \end{cases} \quad (2)$$

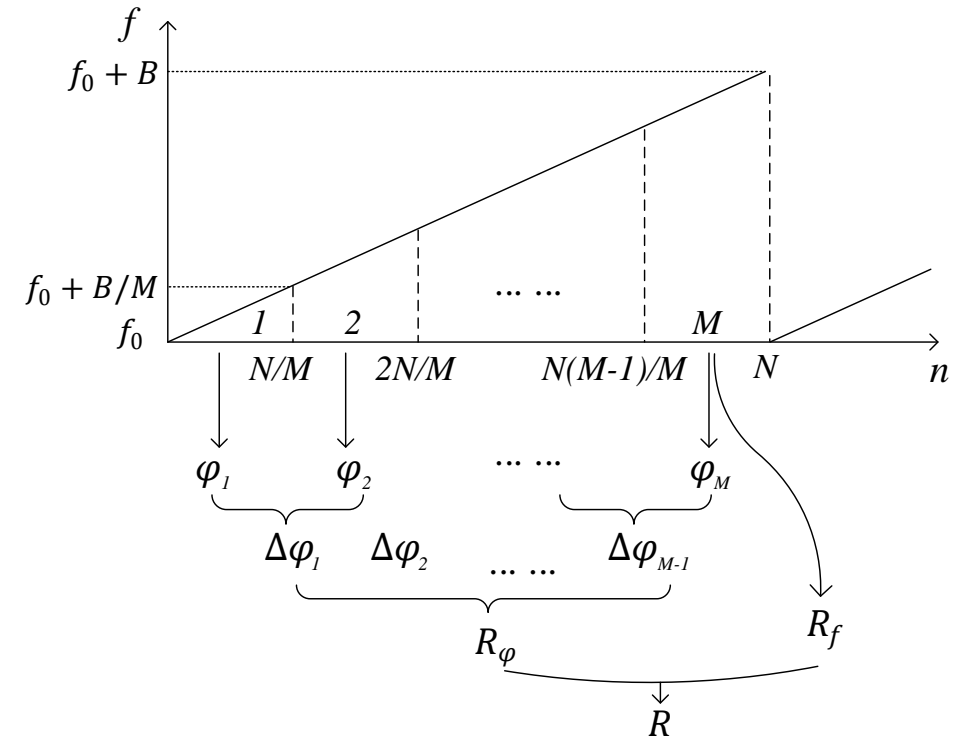
4) Accumulate the phase difference to get: $\Delta\varphi = \Delta\varphi_1 + \Delta\varphi_2 + \dots + \Delta\varphi_{M-1} = \varphi_M - \varphi_1 + p \cdot 2\pi$, where p is the number of 2π that is added in $\Delta\varphi_1 \sim \Delta\varphi_{M-1}$. Therefore, the fine distance with ambiguity can be estimated to be: $R_\varphi = c\Delta\varphi / 4\pi(f_{cM} - f_{c1}) = c\Delta\varphi / 4\pi(M-1)\Delta f$. This is similar to the FSK CW radar with the lower frequency being f_{c1} , the upper frequency being f_{cM} , and unambiguous phase range being $0 \sim 2\pi(M-1)$. Therefore, the unambiguous range of R_φ is $0 \sim c/2\Delta f$, which is the same as the distance resolution ΔR .



5) The absolute fine distance can then be estimated with the combination of R_f and R_φ :

$$R = \begin{cases} R_f + R_\varphi, & F_m[k_0 + 1] \geq F_m[k_0 - 1] \\ R_f + R_\varphi - \Delta R, & F_m[k_0 + 1] < F_m[k_0 - 1] \end{cases} \quad (3)$$

Since R_φ is not frequency resolution limited, the distance R estimated with the proposed technique is not frequency resolution limited.



Noise analysis

The fine distance is estimated as $R_\varphi = c(\varphi_M - \varphi_1 + p \cdot 2\pi)/4\pi(M - 1)\Delta f$, which means the proposed technique's accuracy is directly related to φ_M and φ_1 . The variance of the phase item φ_m under an SNR of η is:

$$\text{Var}(\varphi_m) = \frac{1}{\eta N_1}, \quad (4)$$

where $N_1 = N/M$ is the segment length. Therefore, the variance of $\Delta\varphi$ is:

$$\text{Var}(\Delta\varphi) = \text{Var}(\varphi_M) + \text{Var}(\varphi_1) = \frac{2}{\eta N_1}. \quad (5)$$

Thus, the variance of the R_φ is

$$\text{Var}(R_\varphi) = \frac{c^2}{(4\pi)^2 B^2 (1 - N_1/N)^2} \cdot \frac{2}{\eta N_1}, \quad (6)$$

Noise analysis

It can be seen from (6) that $Var(R_\varphi)$ varies with N_1 . With simple analysis, it can be found that the minimum $Var(R_\varphi)$ is obtained when $N_1 = N/3$, which means $M=3$. The minimum $Var(R_\varphi)$ can then be derived as:

$$Var(R_\varphi) = \frac{27c^2}{32\pi^2\eta NB^2}, \quad (7)$$

which is close to the CRLB of the conventional frequency estimation method [4]:

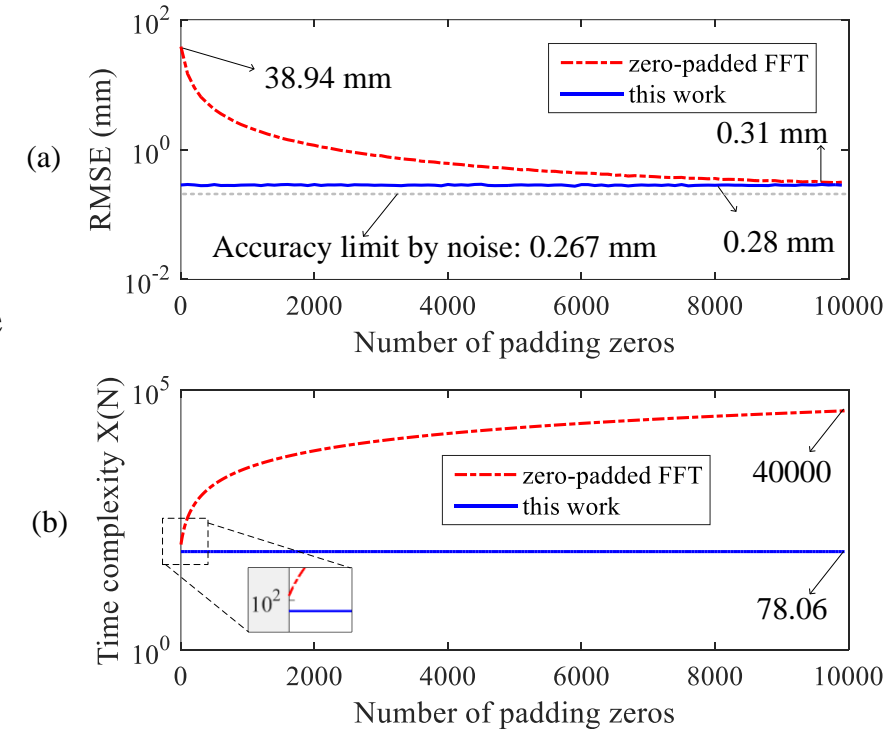
$$Var(R) = \frac{24c^2}{32\pi^2\eta NB^2}. \quad (8)$$

Moreover, with $M=3$, only 3 $N/3$ -points FFT is needed in the proposed technique, which means the time complexity of the proposed technique is only $O(N\log(N/3))$. This is even smaller than the original non-zero-padded FFT's time complexity of $O(N\log(N))$.

Simulations are carried out in MATLAB to validate the proposed technique.

The center frequency is set to 120 GHz. The bandwidth is set to be 4 GHz. The sampling rate is set to be 10 kHz. Additive white Gaussian noise is added to the IF signal with the `awgn()` function in MATLAB. The *SNR* is set to be 20 dB.

Therefore, according to (7) and (8), the accuracy limits of the conventional zero-padded FFT method and the proposed technique are 0.267mm and 0.281mm, respectively.



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Fig. (a) shows the distance estimation root-mean-square- error (RMSE) with the conventional zero-padded FFT method and the proposed technique.

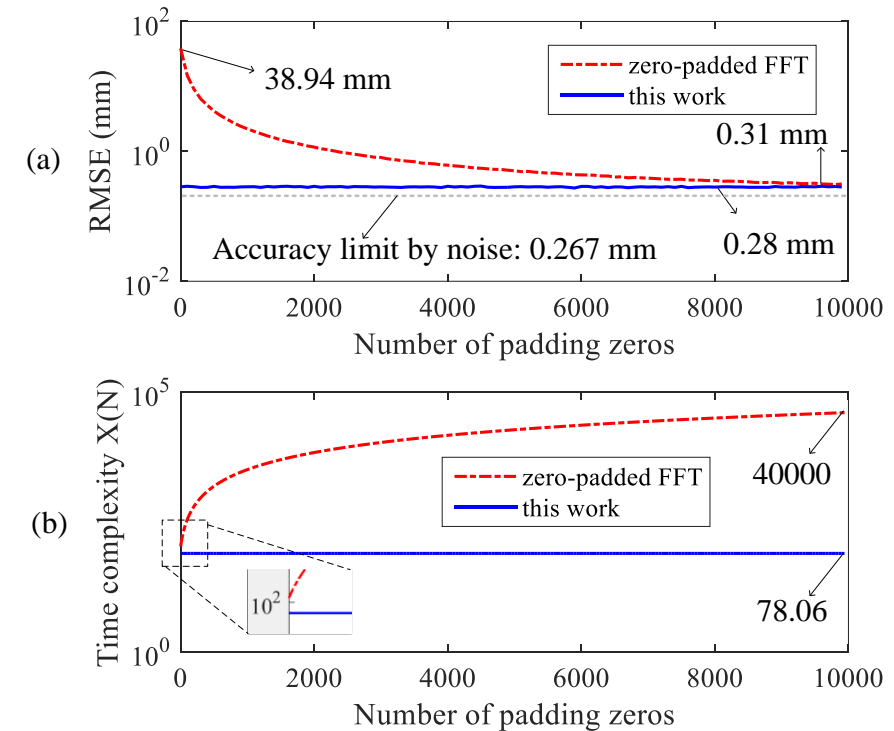
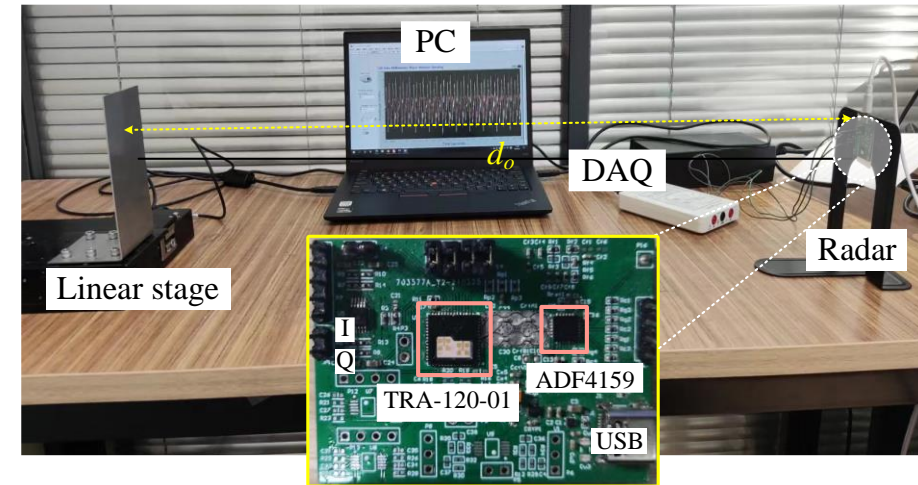


Fig. (b) shows the time complexity of the conventional zero-padded FFT method and the proposed technique.

The experimental setup of distance measurement with a custom-built 120 GHz FMCW radar and a linear-stage Zaber X-LDM060C-AE54D12. The inset shows the detail of the radar.

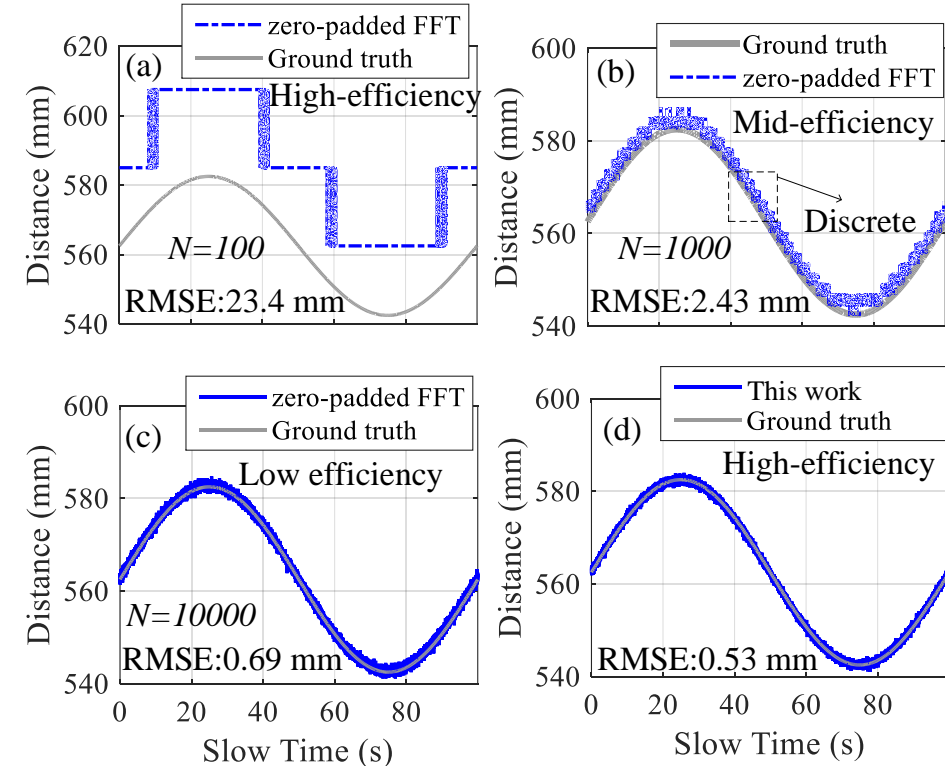


the radar is made with the radar front-end TRA-120-001 (Silicon Radar) and the PLL chip ADF4159. The I/Q signal is sampled by the data acquisition (DAQ) board (National Instruments) for post-signal processing in the computer. The radar sits around $d_o = 56\text{ cm}$ away from the linear stage (Zaber X-LDM060C-AE54D12).

The radar bandwidth is set to 4 GHz, the PRT is set to 6 ms, and the sampling rate is set to 10 kHz. The linear stage is programmed to perform a sinusoidal movement of $2\text{cm}@0.01\text{Hz}$. The slow frequency is to avoid the influence of the Doppler effect.

The measured distance (a)/(b)/(c) with the conventional zero-padded FFT and (c) the proposed technique. In (a)/(b)/(c), the signal is padded to a length of 100/1000/10000.

The Blackman window function is applied to mitigate the influence of the leakage and clutters.



Conclusion

This paper presents a high-efficient and frequency-resolution free PDA technique for accurate distance measurement with FMCW radar.

It achieves similar accuracy of the conventional FFT-based distance estimation method while saving over 500 times of compute resources. The technique is especially suitable for the embedded device which is widely used for radar systems but only has a limited compute resource.

It should be noted that the proposed algorithm is based on the center phase variation, which makes its accuracy only comparable to the frequency evaluation method [5]. By combining the center phase as in [4-5], the accuracy of the proposed technique can be further improved.

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- [3] S. Scherr, S. Ayhan, B. Fischbach, A. Bhutani, M. Pauli and T. Zwick, "An Efficient Frequency and Phase Estimation Algorithm With CRB Performance for FMCW Radar Applications," *IEEE Transactions on Instrumentation and Measurement*, vol. 64, no. 7, pp. 1868-1875, July 2015.
- [4] L. Piotrowsky, T. Jaeschke, S. Kueppers, J. Siska and N. Pohl, "Enabling High Accuracy Distance Measurements With FMCW Radar Sensors," *IEEE Transactions on Microwave Theory and Techniques*, vol. 67, no. 12, pp. 5360-5371, Dec. 2019.
- [5] M. Scherhauf, F. Hammer, M. Pichler-Scheder, C. Kastl and A. Stelzer, "Radar Distance Measurement With Viterbi Algorithm to Resolve Phase Ambiguity," *IEEE Transactions on Microwave Theory and Techniques*, vol. 68, no. 9, pp. 3784-3793, Sept. 2020.

Thank you!