

WE3E-319-XB39

Vector Single-Source SIE Formulation for Scattering Analysis of Multilayered Object

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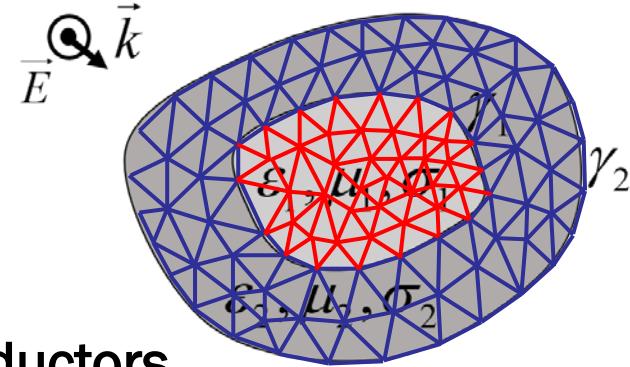
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Outline

- 1 • State-of-the-art
- 2 • The Proposed Approach
- 3 • Numerical Results
- 4 • Conclusions

- **Volumetric mesh based methods**
 - FDTD, FEM, ...
 - ✓ Includes proximity and skin effects
 - ✗ **Slow:** mesh the surrounding medium & the whole volume of the conductors

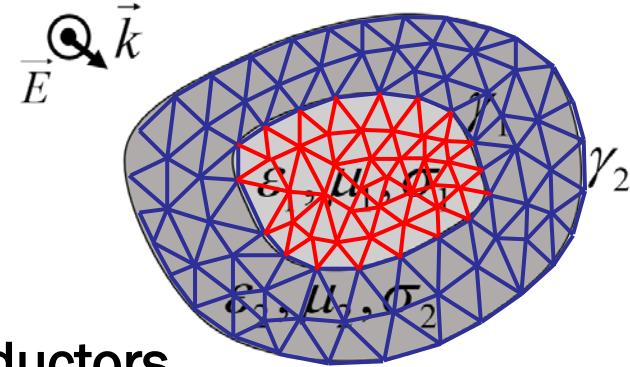


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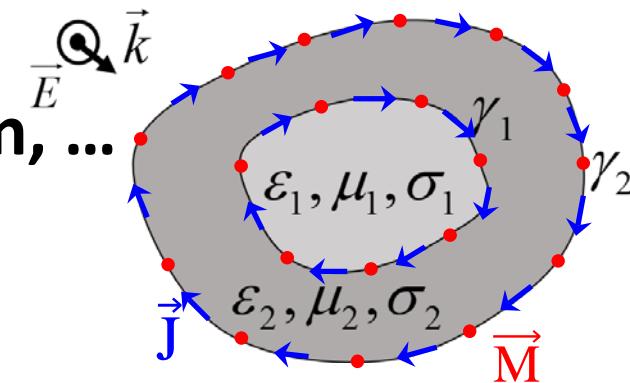


- **Surface mesh based methods**

- PMCHWT formulation, Combined tangential formulation, ...

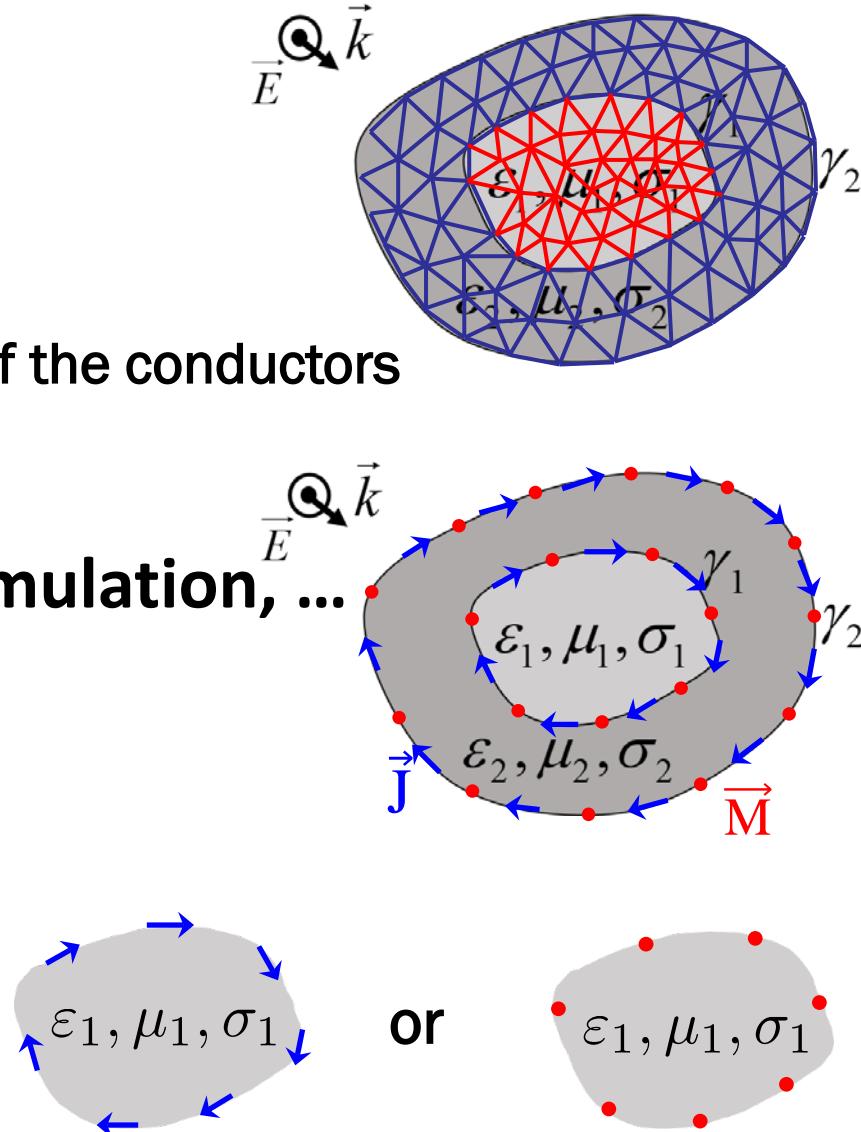
✓ Unknowns on the surface

✗ Two region problems for many interfaces

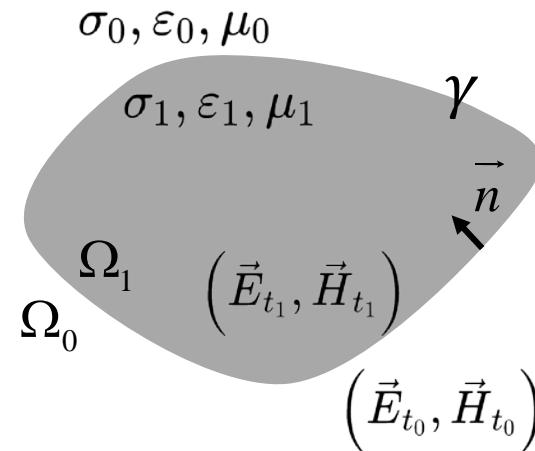


- **Volumetric mesh based methods**
 - FDTD, FEM, ...
 - ✓ Includes proximity and skin effects
 - ✗ Slow: mesh the surrounding medium & the whole volume of the conductors

- **Surface mesh based methods**
 - PMCHWT formulation, Combined tangential formulation, ...
 - ✓ Unknowns on the surface
 - ✗ Two region problems for many interfaces
 - Single-source(SS) formulations: SS-SVS, GIBC, ...
 - ✓ Single source → more efficient
 - ✗ Objects are embedded in multilayers



- Surface equivalence theorem

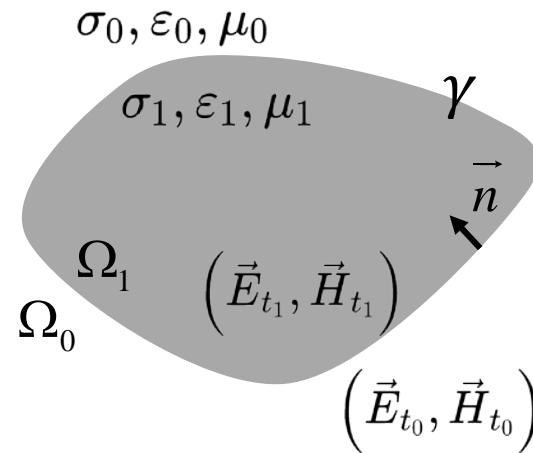


Original model

$$0 = \vec{H}_{t_1} - \vec{H}_{t_0}$$

$$0 = \vec{E}_{t_1} - \vec{E}_{t_0}$$

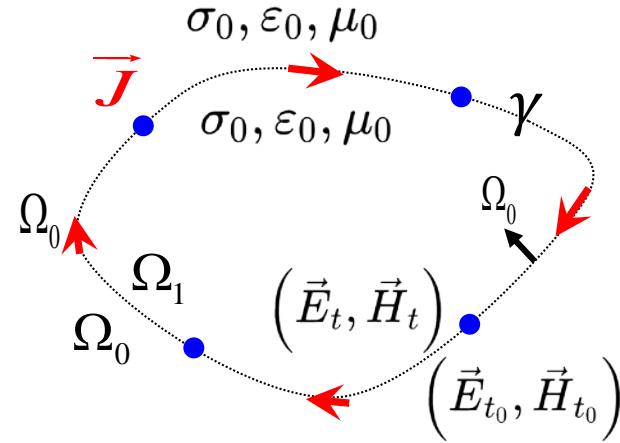
- Surface equivalence theorem



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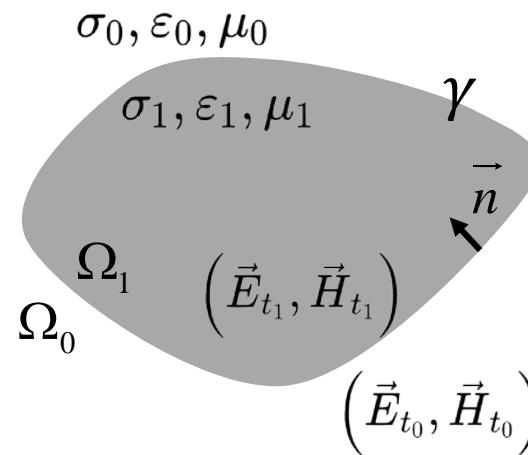


dual-source model

$$\vec{J} = \vec{H}_t - \vec{H}_{t_0}$$

$$\vec{M} = \vec{E}_t - \vec{E}_{t_0}$$

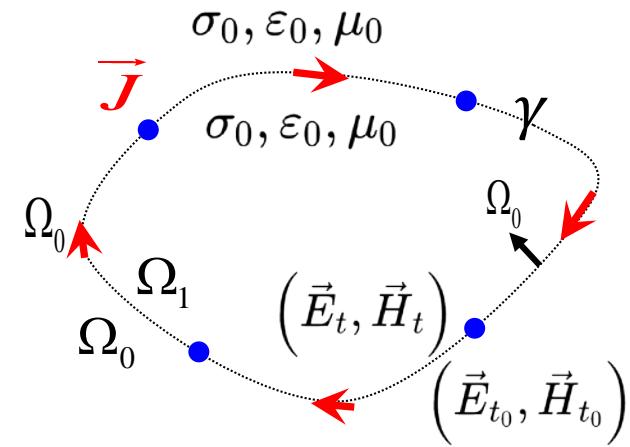
- Surface equivalence theorem



Original model

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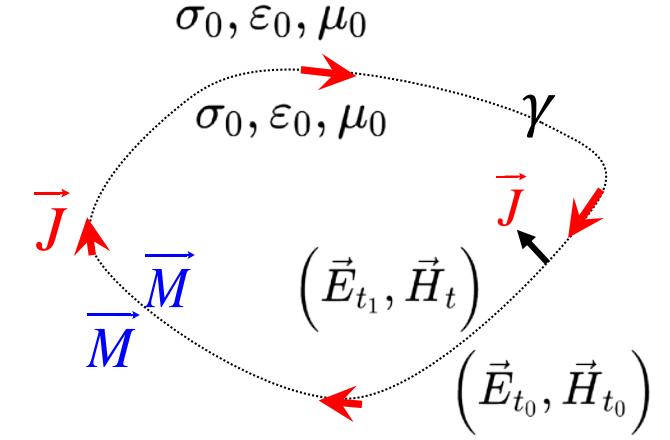
$$0 = \vec{E}_{t_1} - \vec{E}_{t_0}$$



dual-source model

$$\vec{J} = \vec{H}_t - \vec{H}_{t_0}$$

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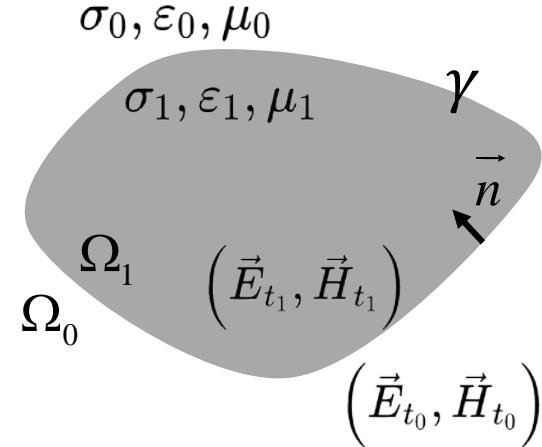
single-source model

$$\vec{J} = \vec{H}_t - \vec{H}_{t_1}$$

$$0 = \vec{E}_t - \vec{E}_{t_0}$$

enforce $\vec{E}_t|_{\vec{r} \in \gamma} = \vec{E}_{t_1}|_{\vec{r} \in \gamma}$

- A single penetrable object
 - Original model
 - **Stratton-Chu formulation** for a homogenous object



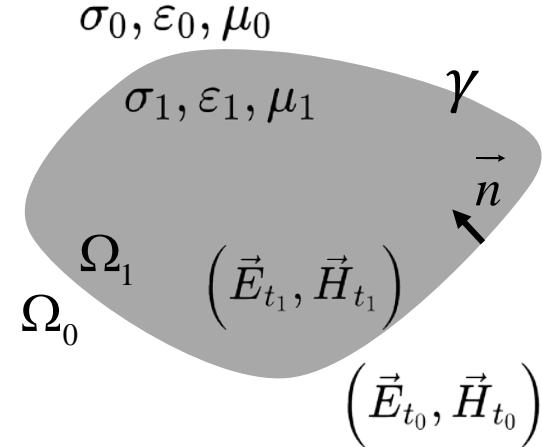
$$\frac{1}{2} \vec{n} \times \vec{n} \times \vec{E} = \vec{n} \times \vec{n} \times \mathcal{L} [\vec{n}' \times \vec{H}] + \vec{n} \times \vec{n} \times \mathcal{K} [\vec{n}' \times \vec{E}]$$

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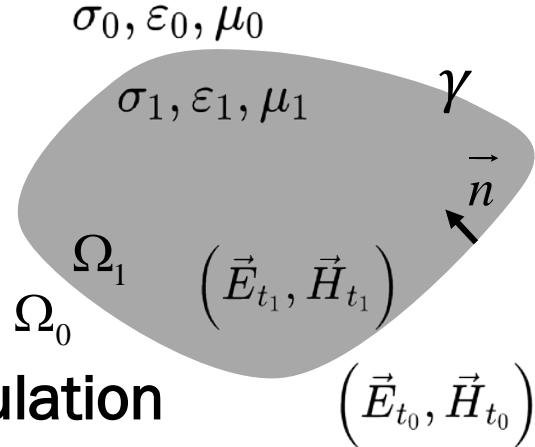
- Discretize the boundary with the basis function

$$\vec{n}' \times \vec{E}(\vec{r}') = \sum_{n=1}^N e_n \vec{n}' \times f_n(\vec{r}') \quad \vec{n}' \times \vec{H}(\vec{r}') = \sum_{n=1}^N h_n f_n(\vec{r}')$$



Proposed Approach

- A single penetrable object
 - Original model
 - The **Galerkin scheme** is used to test the Stratton-Chu formulation

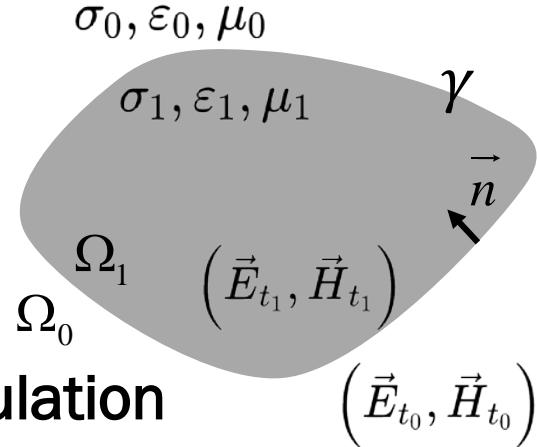


$$\mathbf{H} = \underbrace{[\mathbf{L}]^{-1} \left[\frac{1}{2} \mathbf{U} - \mathbf{K} \right]}_{\mathbf{Y}} \mathbf{E}$$

$$\begin{aligned} [\mathbf{U}]_{mn} &= -\langle \mathbf{f}_m, \mathbf{n} \times \mathbf{n} \times (\mathbf{f}_n) \rangle \\ [\mathbf{L}]_{mn} &= -\langle \mathbf{f}_m, \mathbf{n} \times \mathbf{n} \times (\mathcal{L}(\mathbf{f}_n)) \rangle \\ [\mathbf{K}]_{mn} &= -\langle \mathbf{f}_m, \mathbf{n} \times \mathbf{n} \times (\mathcal{K}[\mathbf{n}' \times \mathbf{f}_n]) \rangle \end{aligned}$$

Proposed Approach

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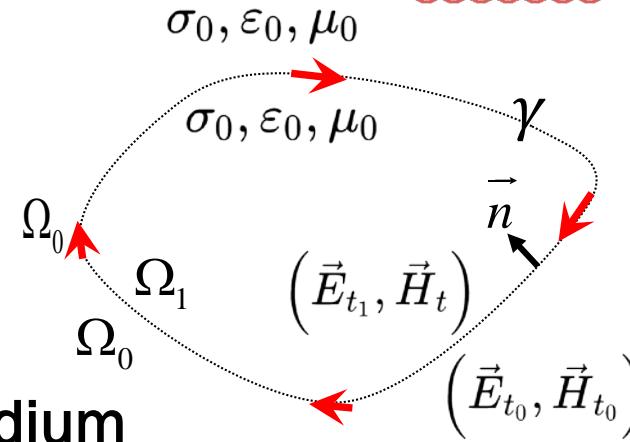
$$\mathbf{H} = \underbrace{[\mathbf{L}]^{-1} \left[\frac{1}{2} \mathbf{U} - \mathbf{K} \right]}_{\mathbf{Y}} \mathbf{E}$$

- \mathbf{Y} is defined as the **surface admittance operator**

$$\begin{aligned} [\mathbf{U}]_{mn} &= -\langle \mathbf{f}_m, \mathbf{n} \times \mathbf{n} \times (\mathbf{f}_n) \rangle \\ [\mathbf{L}]_{mn} &= -\langle \mathbf{f}_m, \mathbf{n} \times \mathbf{n} \times (\mathcal{L}(\mathbf{f}_n)) \rangle \\ [\mathbf{K}]_{mn} &= -\langle \mathbf{f}_m, \mathbf{n} \times \mathbf{n} \times (\mathcal{K}[\mathbf{n}' \times \mathbf{f}_n]) \rangle \end{aligned}$$

Proposed Approach

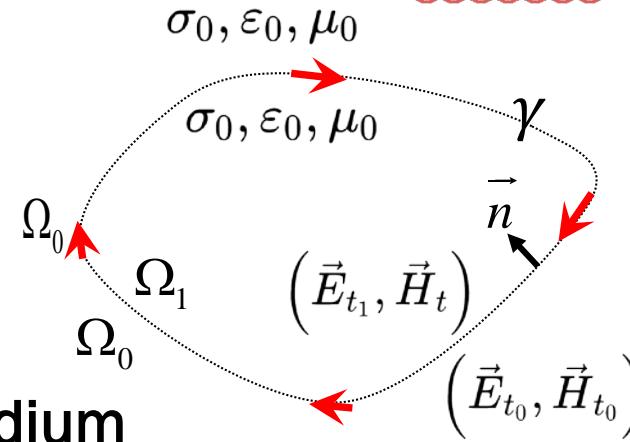
- A single penetrable object
 - Equivalent model
 - The penetrable object is replaced by the background medium



$$\hat{\mathbf{H}} = \underbrace{[\hat{\mathbf{L}}]^{-1} \left[\frac{1}{2} \hat{\mathbf{U}} - \hat{\mathbf{K}} \right]}_{\hat{\mathbf{Y}}} \mathbf{E}$$

Proposed Approach

- A single penetrable object
 - Equivalent model
 - The penetrable object is replaced by the background medium



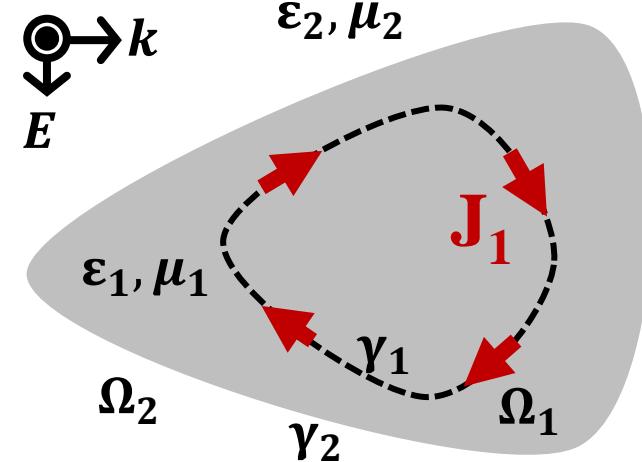
$$\hat{\mathbf{H}} = \underbrace{\left[\hat{\mathbf{L}}^{-1} \left[\frac{1}{2} \hat{\mathbf{U}} - \hat{\mathbf{K}} \right] \right]}_{\hat{\mathbf{Y}}} \mathbf{E}$$

- \mathbf{Y}_s is defined as the **differential** surface admittance operator (DSAO)

$$\mathbf{J} = \hat{\mathbf{H}} - \mathbf{H} = \underbrace{(\hat{\mathbf{Y}} - \mathbf{Y})}_{\mathbf{Y}_s} \mathbf{E}$$

Proposed Approach

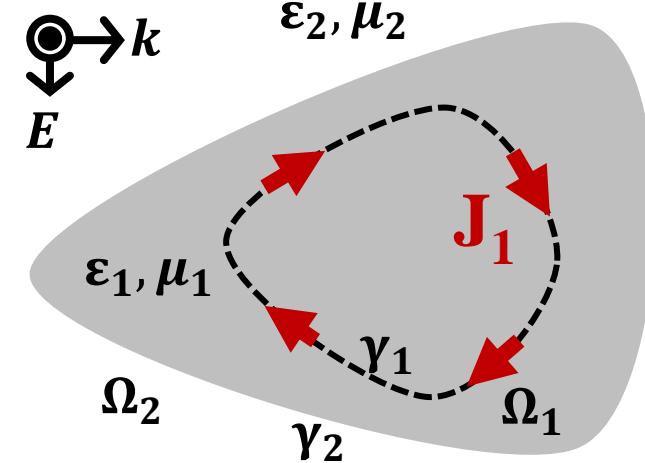
- Two-layered object
 - Original model
 - Inhomogeneous Stratton-Chu formulation



$$T \mathbf{n}_2 \times \mathbf{E} = \mathbf{n}_2 \times \mathcal{L}_2 [\mathbf{n}'_2 \times \mathbf{H}_2] + \mathbf{n}_2 \times \mathcal{K}_2 [\mathbf{n}'_2 \times \mathbf{E}_2] + \mathbf{n}_1 \times \mathcal{L}_1 [\mathbf{J}_1]$$

Proposed Approach

- Two-layered object
 - Original model
 - Inhomogeneous Stratton-Chu formulation



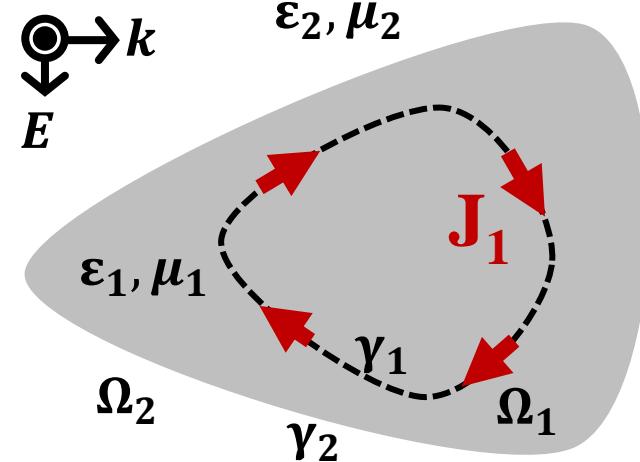
$$T \mathbf{n}_2 \times \mathbf{E} = \mathbf{n}_2 \times \mathcal{L}_2 [\mathbf{n}'_2 \times \mathbf{H}_2] + \mathbf{n}_2 \times \mathcal{K}_2 [\mathbf{n}'_2 \times \mathbf{E}_2] + \boxed{\mathbf{n}_1 \times \mathcal{L}_1 [\mathbf{J}_1]}$$



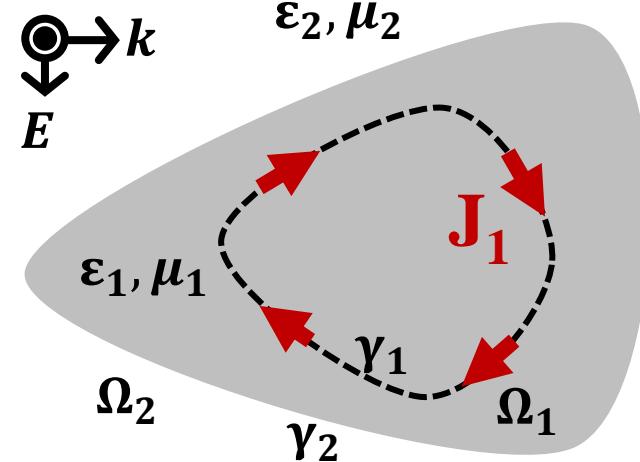
Additional term caused by the current from the first equivalence

Proposed Approach

- Two-layered object
 - Original model
 - Test the formulation on γ_1
 - Test the formulation on γ_2



- Two-layered object
 - Original model
 - Test the formulation on Υ_1

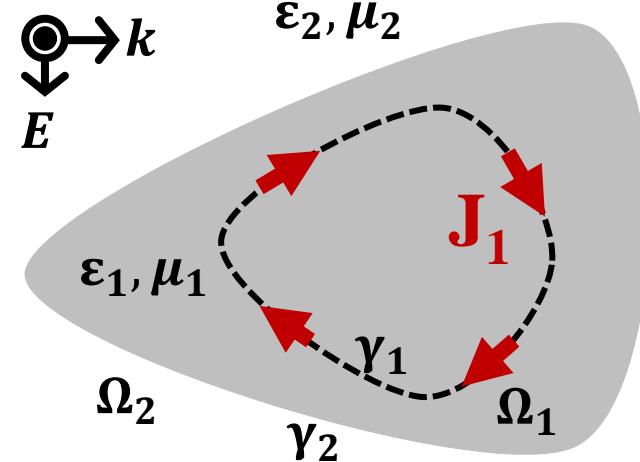


$$\mathbf{U}_{(1,1)}^2 \mathbf{E}_1 = \mathbf{L}_{(2,1)}^2 \mathbf{H}_2 + \mathbf{K}_{(2,1)}^2 \mathbf{E}_2 + \mathbf{L}_{(1,1)}^2 \mathbf{J}_1$$

- Test the formulation on Υ_2

$$\frac{1}{2} \mathbf{U}_{(2,2)}^2 \mathbf{E}_2 = \mathbf{L}_{(2,2)}^2 \mathbf{H}_2 + \mathbf{K}_{(2,2)}^2 \mathbf{E}_2 + \mathbf{L}_{(1,2)}^2 \mathbf{J}_1$$

- Two-layered object
 - Original model
 - Test the formulation on γ_1



$$\mathbf{U}_{(1,1)}^2 \mathbf{E}_1 = \mathbf{L}_{(2,1)}^2 \mathbf{H}_2 + \mathbf{K}_{(2,1)}^2 \mathbf{E}_2 + \mathbf{L}_{(1,1)}^2 \mathbf{J}_1$$

- Test the formulation on γ_2

$$\frac{1}{2} \mathbf{U}_{(2,2)}^2 \mathbf{E}_2 = \mathbf{L}_{(2,2)}^2 \mathbf{H}_2 + \mathbf{K}_{(2,2)}^2 \mathbf{E}_2 + \mathbf{L}_{(1,2)}^2 \mathbf{J}_1$$

$$\mathbf{H}_2 = \mathbf{Y}_2 \mathbf{E}_2$$

Proposed Approach

- Two-layered object
 - Original model

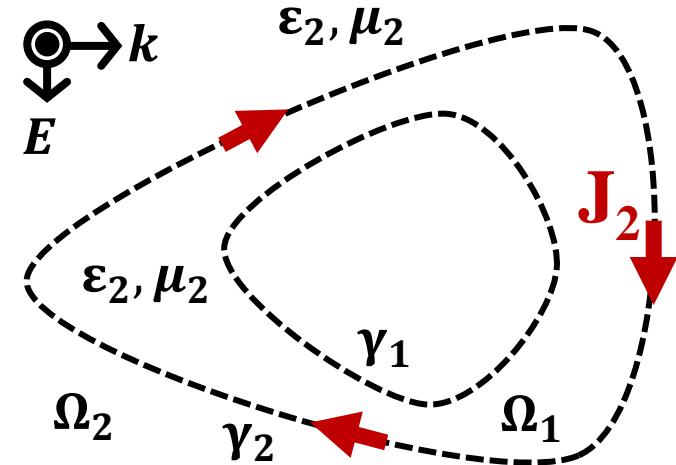
$$\mathbf{H}_2 = \mathbf{Y}_2 \mathbf{E}_2$$

$$\mathbf{J} = \widehat{\mathbf{H}} - \mathbf{H} = \underbrace{(\widehat{\mathbf{Y}} - \mathbf{Y})}_{\mathbf{Y}_s} \mathbf{E}$$

$$\mathbf{H}_2 = \underbrace{\left[\mathbf{L}_{(2,2)}^2 + \mathbf{L}_{(1,2)}^2 \mathbf{Y}_{S_{\gamma_1}} \mathbf{C}_{\gamma_1} \mathbf{L}_{(2,1)}^2 \right]^{-1}}_{\mathbf{Y}_2} \underbrace{\left[\frac{1}{2} \mathbf{U}_{(2,2)}^2 - \mathbf{K}_{(2,2)}^2 - \mathbf{L}_{(1,2)}^2 \mathbf{Y}_{S_{\gamma_1}} \mathbf{C}_{\gamma_1} \mathbf{K}_{(2,1)}^2 \right]}_{\mathbf{E}_2} \mathbf{E}_2$$

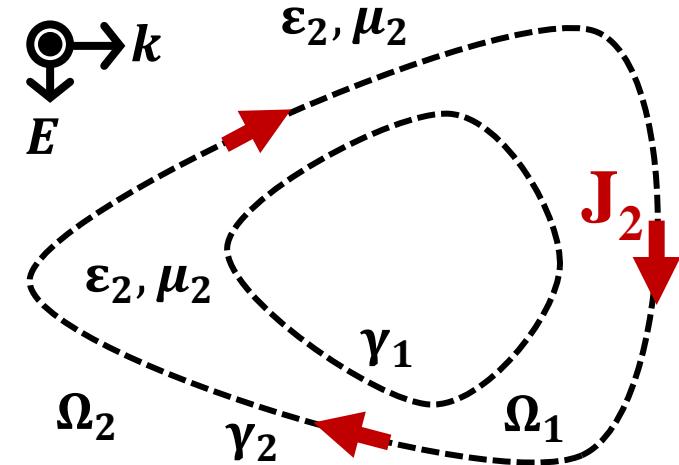
$$\left(\mathbf{U}_{(1,1)} - \mathbf{L}_{(1,1)}^{(2)} \mathbf{Y}_{s_{\gamma_1}} \right)^{-1}$$

- Two-layered object
 - Equivalent model
 - Test the formulation on Υ_2



$$\frac{1}{2} \mathbf{U}_{(2,2)} \widehat{\mathbf{E}}_2 = \widehat{\mathbf{L}}_{(2,2)}^{(2)} \widehat{\mathbf{H}}_2 + \widehat{\mathbf{K}}_{(2,2)}^{(2)} \widehat{\mathbf{E}}_2 \quad \Rightarrow \quad \widehat{\mathbf{H}}_2 = \widehat{\mathbf{Y}}_2 \widehat{\mathbf{E}}_2$$

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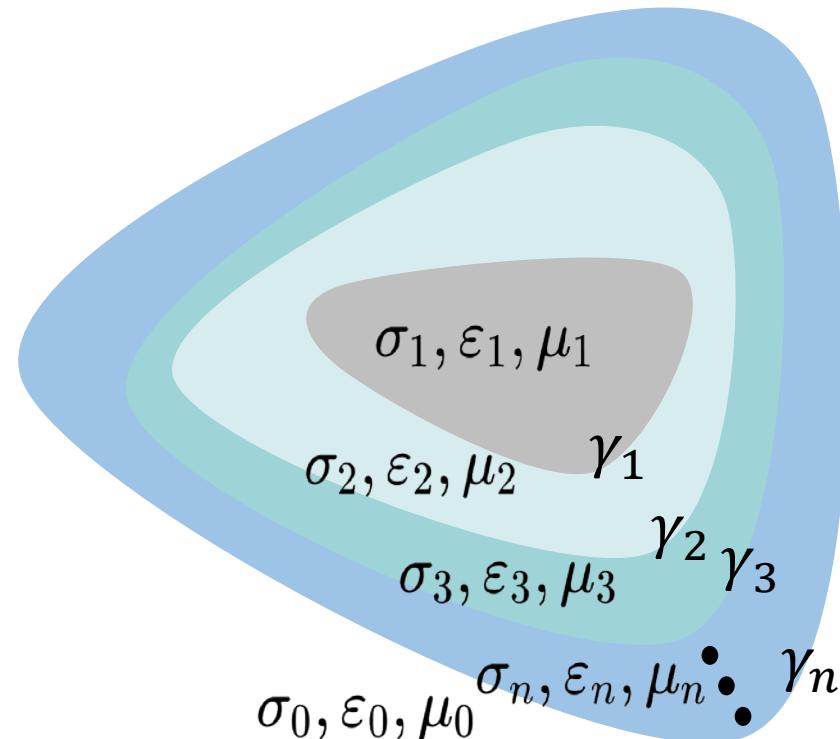
- Combine the SAO in the original model

$$\mathbf{H}_2 = \mathbf{Y}_2 \mathbf{E}_2$$

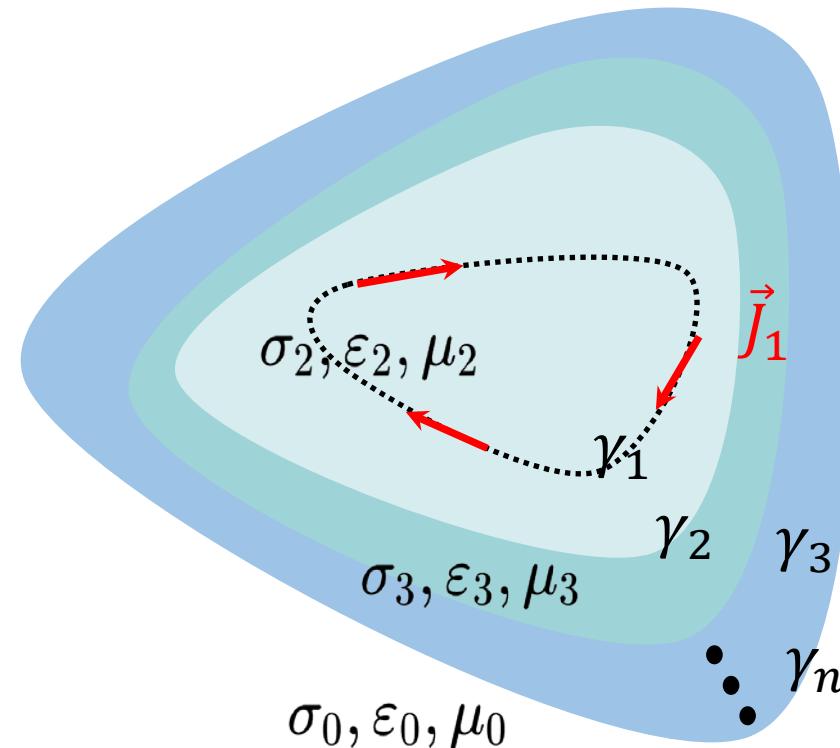
$$\mathbf{J}_2 = \widehat{\mathbf{H}}_2 - \mathbf{H}_2 = (\widehat{\mathbf{Y}}_2 - \mathbf{Y}_2) \mathbf{E}_2 = \mathbf{Y}_{\gamma_2} \mathbf{E}_2$$

Proposed Approach

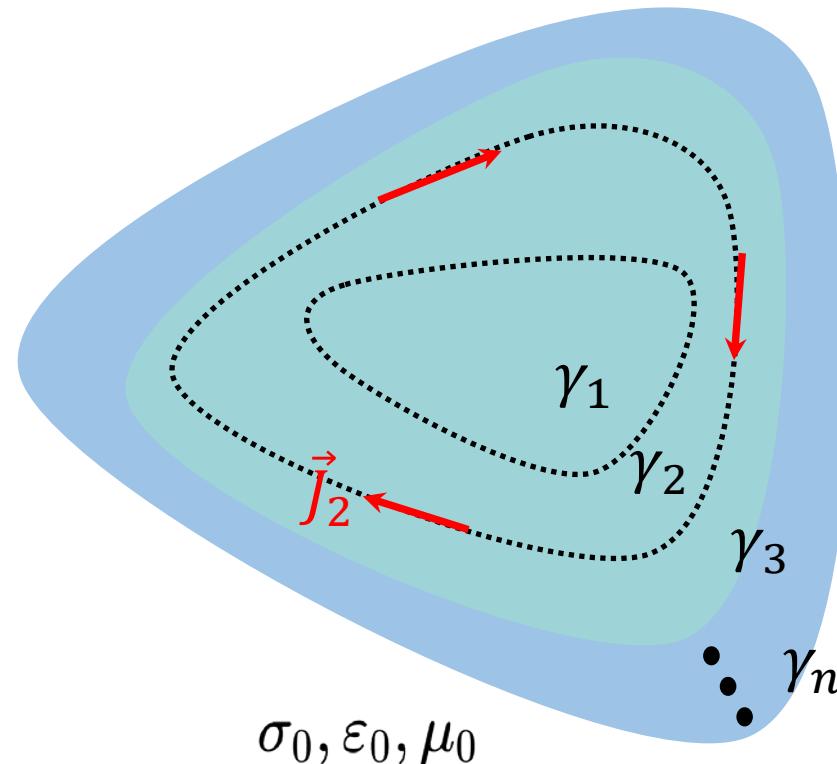
- Multi-layered object
 - Original model



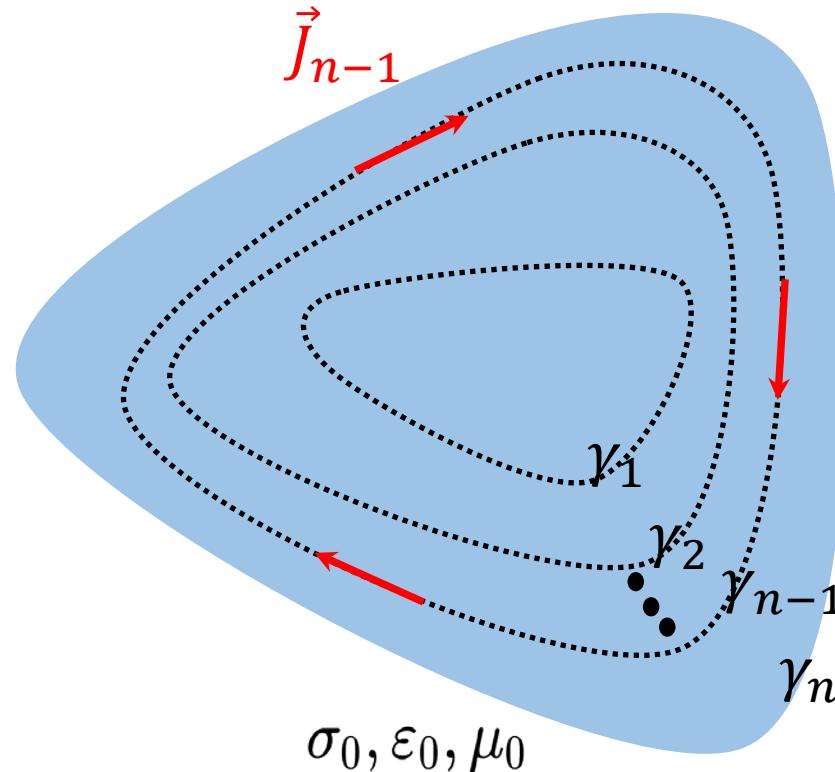
- Multi-layered object
 - 1st equivalent model



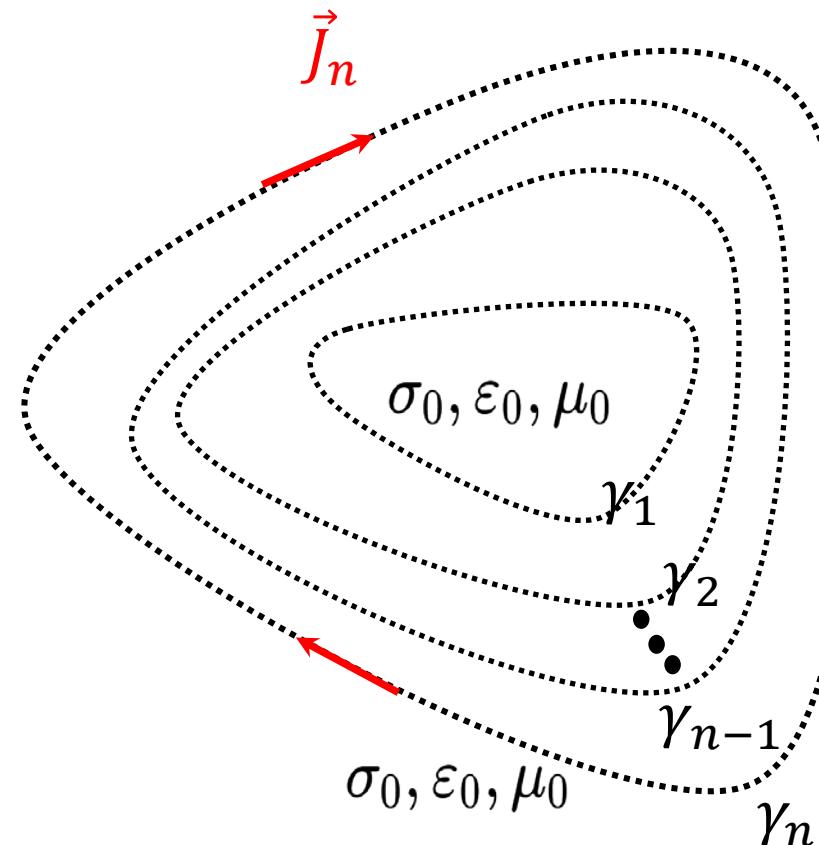
- Multi-layered object
 - 2nd equivalent model



- Multi-layered object
 - $(n-1)^{\text{th}}$ equivalent model



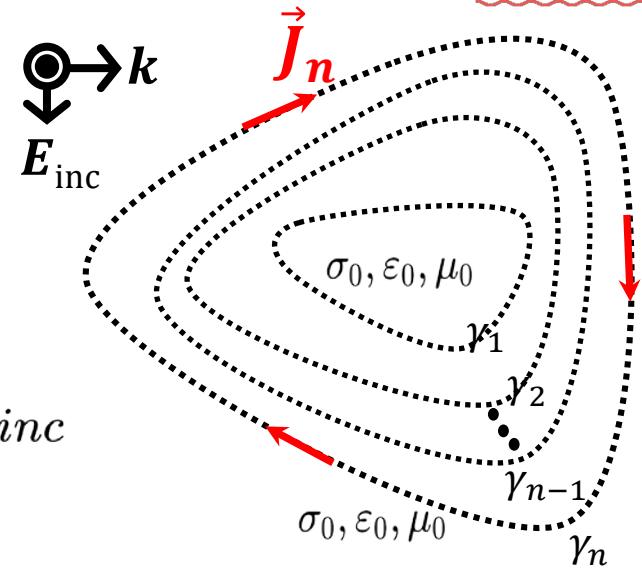
- Multi-layered object
 - n^{th} equivalent model



Proposed Approach

- Outer problem
 - Electric field integral equation (EFIE)

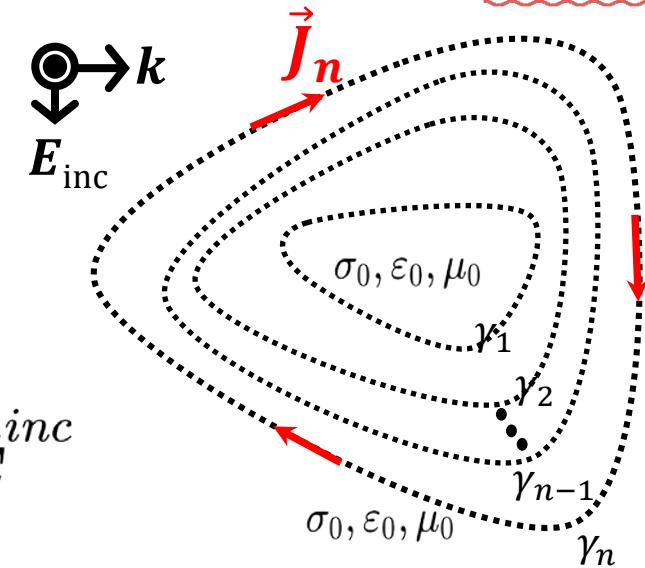
$$\vec{E}(\vec{r}) = -j\omega\mu \int_{\gamma} \left(1 + \frac{1}{k_0^2} \nabla' \nabla' \cdot \right) \vec{J}(\vec{r}') G_0(\vec{r}, \vec{r}') d\vec{r}' + \vec{E}^{inc}$$



Proposed Approach

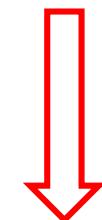
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Discretization and testing

$$\mathbf{J} = \mathbf{Y}_S \mathbf{E}$$

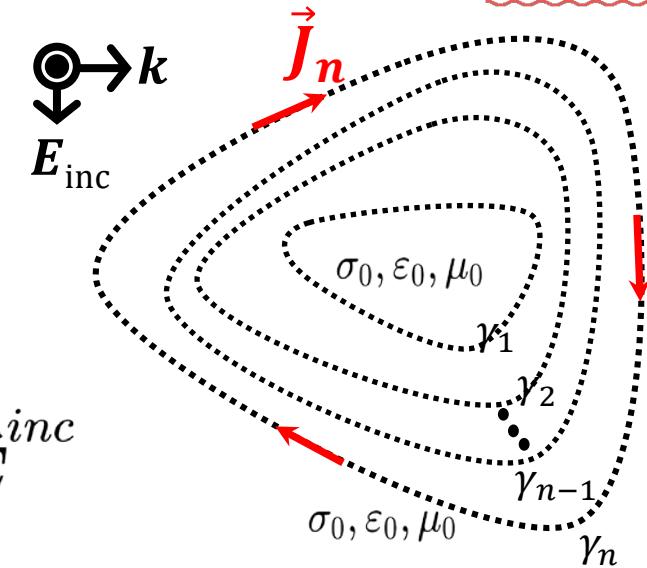


$$\mathbf{U}\mathbf{E} = \mathbf{L}\mathbf{Y}_S \mathbf{E} + \mathbf{E}^i$$

Proposed Approach

- Outer problem
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$$\vec{E}(\vec{r}) = -j\omega\mu \int_{\gamma} \left(1 + \frac{1}{k_0^2} \nabla' \nabla' \cdot \right) \vec{J}(\vec{r}') G_0(\vec{r}, \vec{r}') d\vec{r}' + \vec{E}^{inc}$$



Discretization and testing

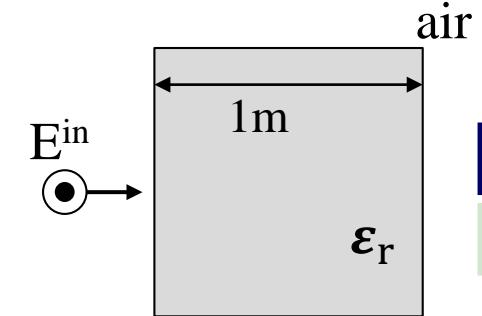
$$\mathbf{J} = \mathbf{Y}_S \mathbf{E}$$

$$\mathbf{U}\mathbf{E} = \mathbf{L}\mathbf{Y}_S \mathbf{E} + \mathbf{E}^i$$

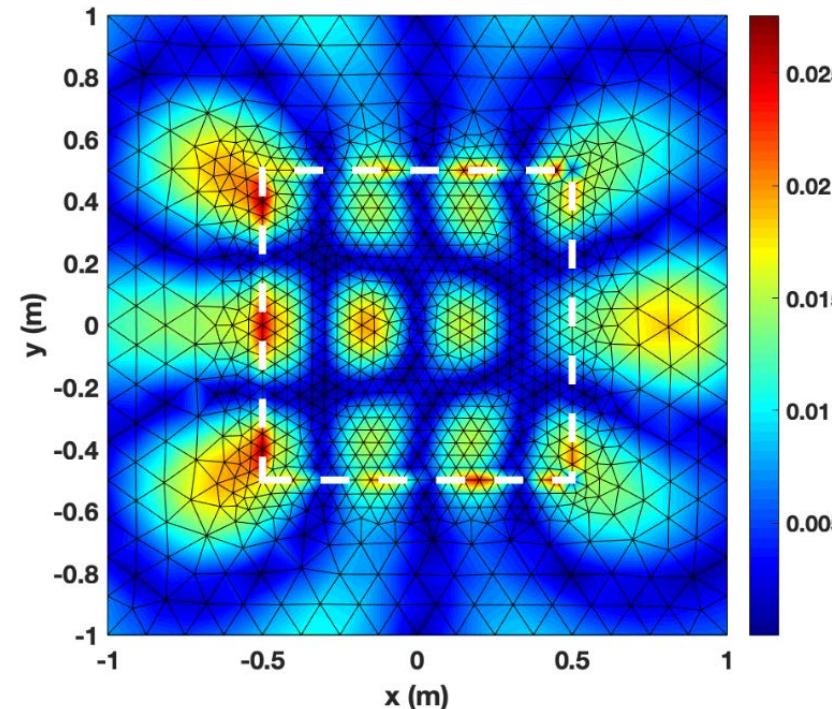
$$\mathbf{E} = [\mathbf{U} - \mathbf{L}\mathbf{Y}_S]^{-1} \mathbf{E}^i$$

Example

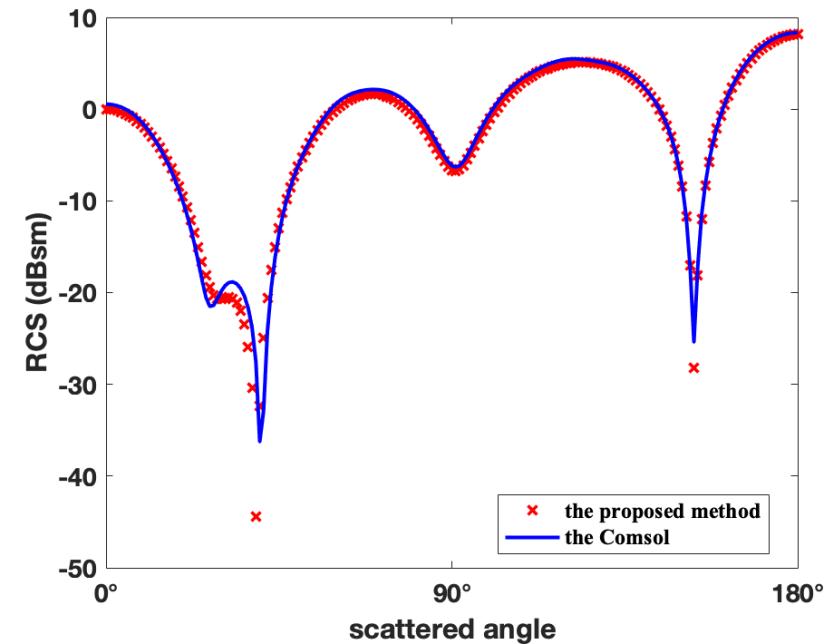
- TM mode-Dielectric cuboid object



Frequency	Mesh size	ϵ_r
300MHz	0.1 m	4



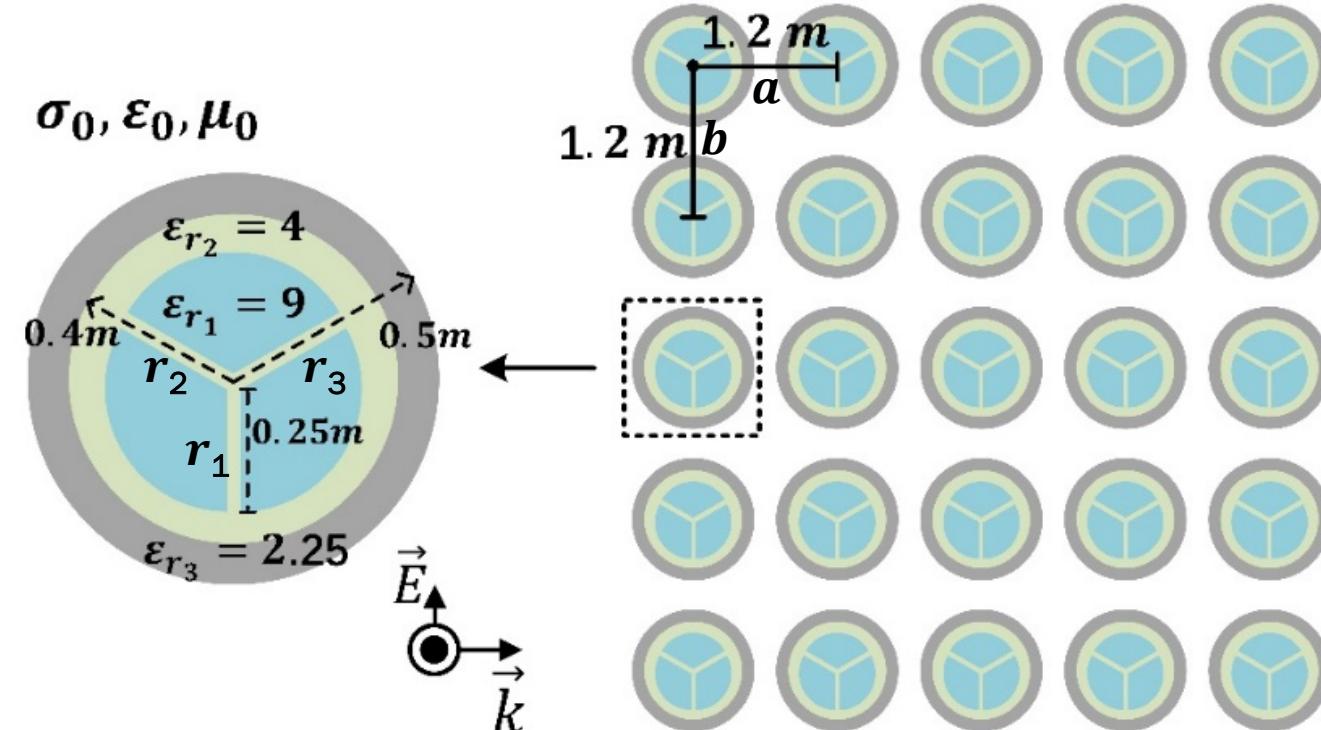
a) Relative error of electric filed



b) Radar cross section

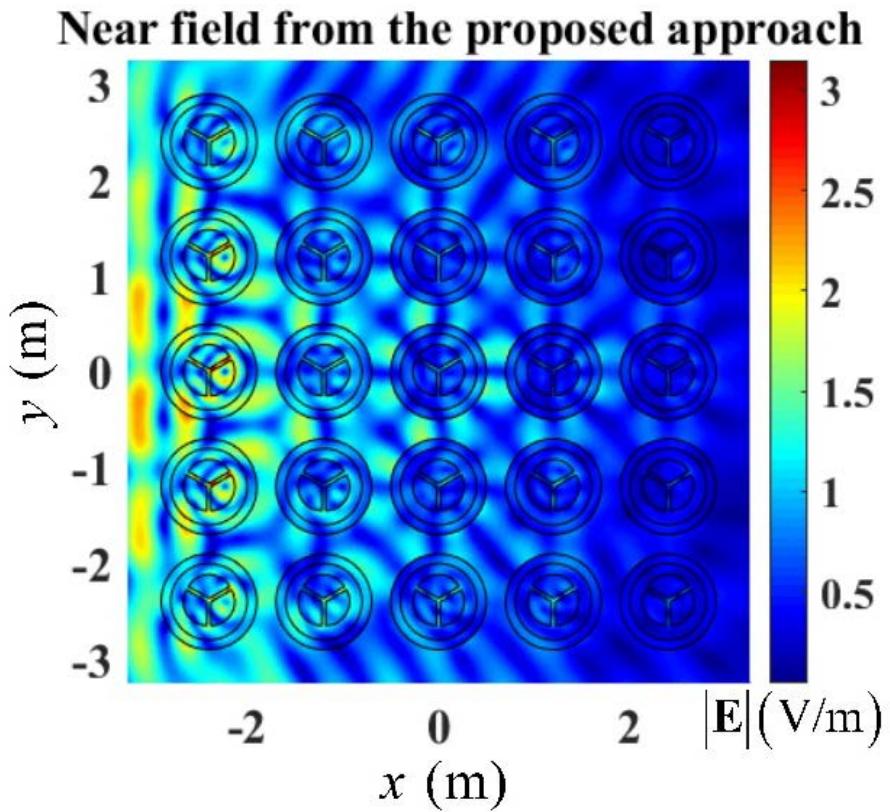
- TE mode-3-layered cylindrical object

Frequency	Mesh size	r_1	r_2	r_3	$a = b$	ϵ_{r1}	ϵ_{r2}	ϵ_{r3}
300MHz	0.03 m	0.25 m	0.4 m	0.5 m	1.2 m	9	4	2.25

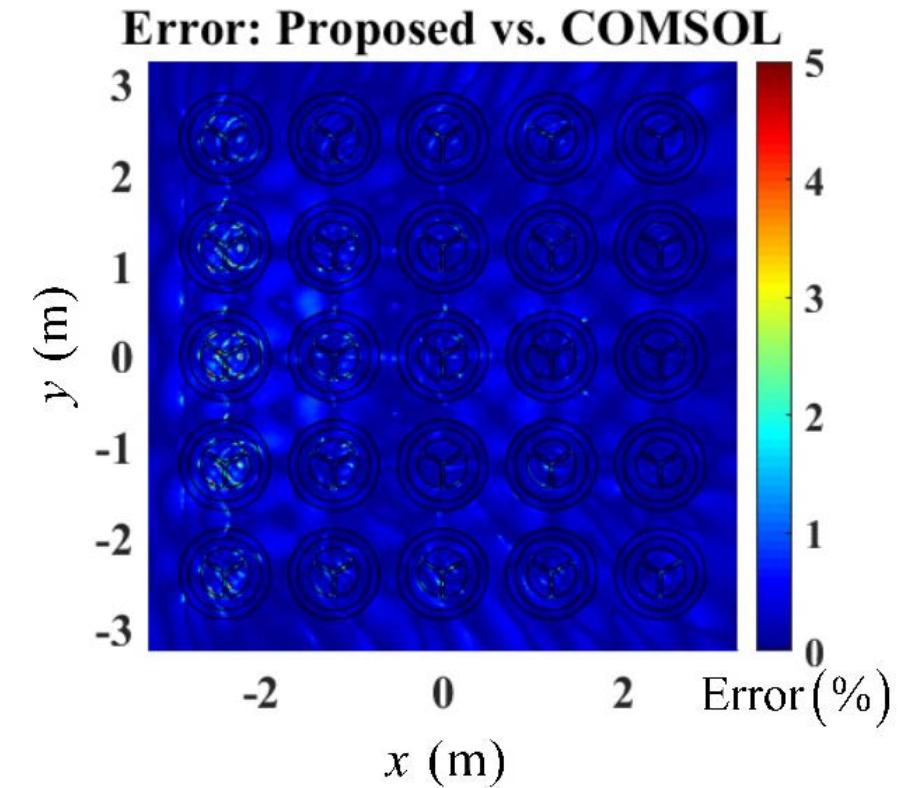


Example

- TE mode-3-layered cylindrical object

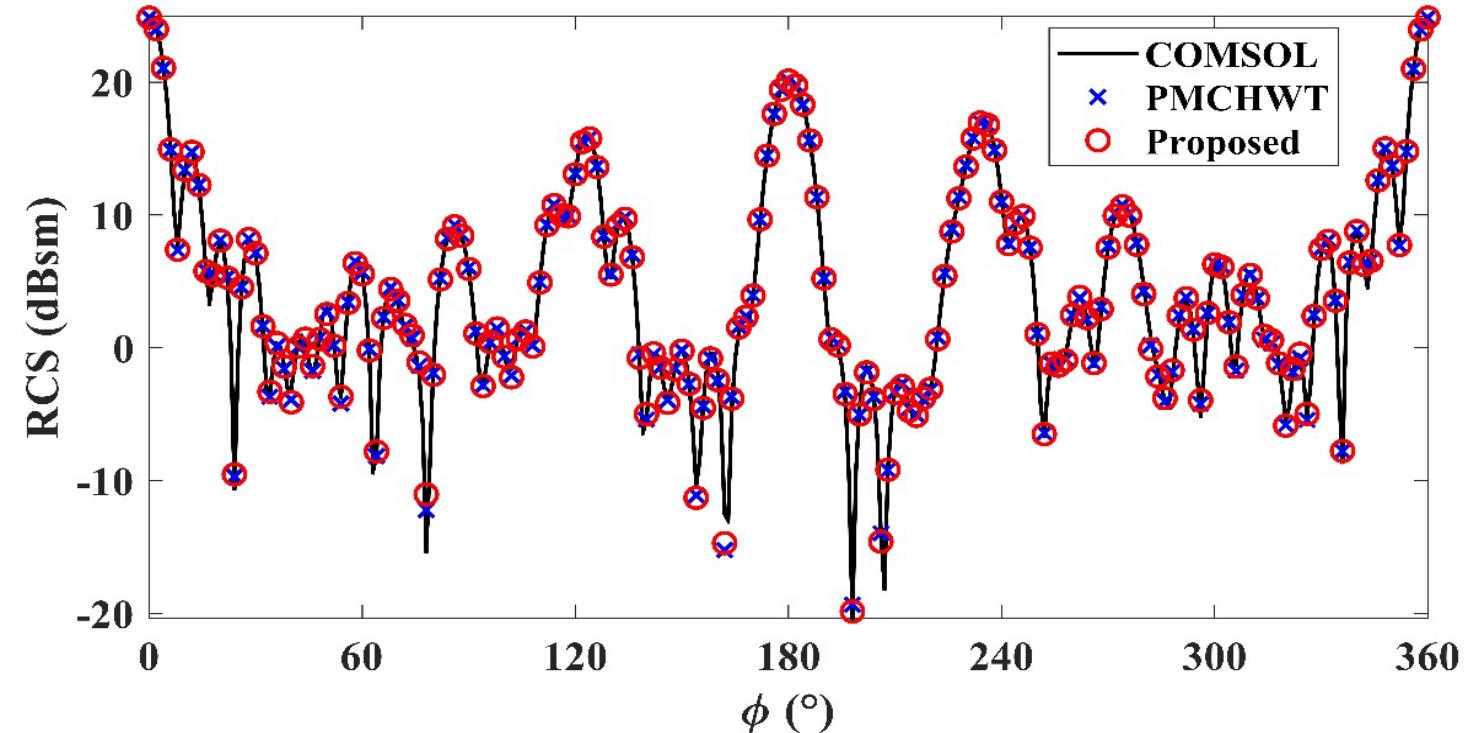


a) Near field from the proposed approach



b) Relative error over the COMSOL

- TE mode-3-layered cylindrical object



c) Radar cross section obtained from the COMSOL, PMCHWT formulation, and the proposed approach

- TE mode-3-layered cylindrical object

	Proposed	PMCHWT	Ratio
Total Time [s]	403	1,236	0.33
Generating YS [s]	2	-	-
Filling time [s]	399	1,193	0.33
Solving time [s]	2	43	0.05
Memory consumption [MB]	770	4,857	0.16
Condition number	424	117,550	-
Overall count of unknowns	2600	14,800	0.18

d) computational cost compared with the PMCHWT formulation

- A novel single source SIE for electromagnetic analysis by multilayer embedded objects is proposed.

- **Fast**
- **Accurate**
- **Single electric current source**
- **Unknowns only on outermost boundary**
- **Significant less count of unknowns compared with other SIEs**

Q&A