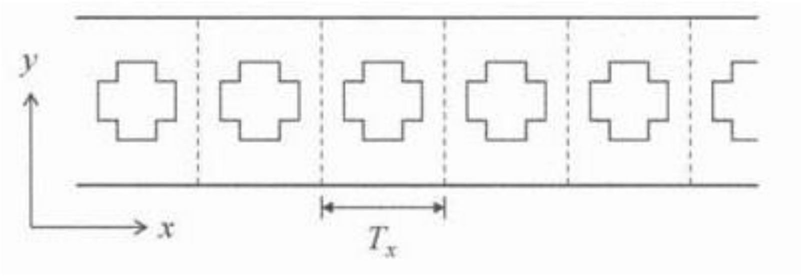
We3E-5

Normal Incidence Scattering of Waveguide-Like FSS/PSS in Scalar 2D-FEM/MM Extracting the Frequency dependence

G. Garcia-Contreras, J. Córcoles, J. A. Ruiz-Cruz
Universidad Autónoma de Madrid, Madrid, Spain

Introduction

- Frequency/Polarization Selective Surfaces (FSS/PSS)
  - 2D periodic structures based on a Unit Cell



1D Periodic Structure [1]

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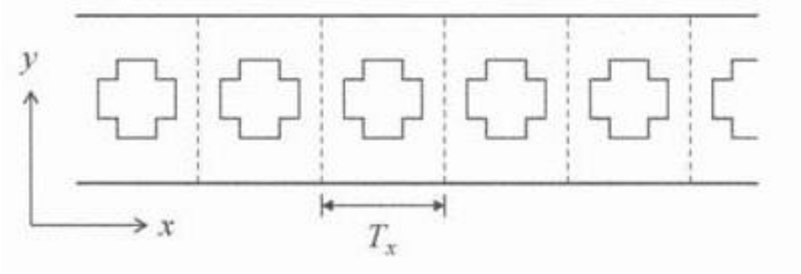

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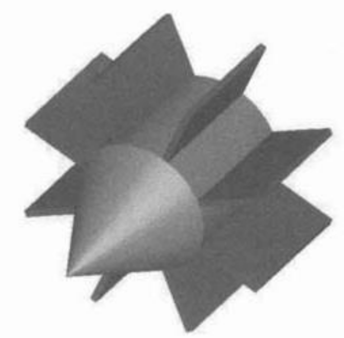

Angular Periodic Structure [2]

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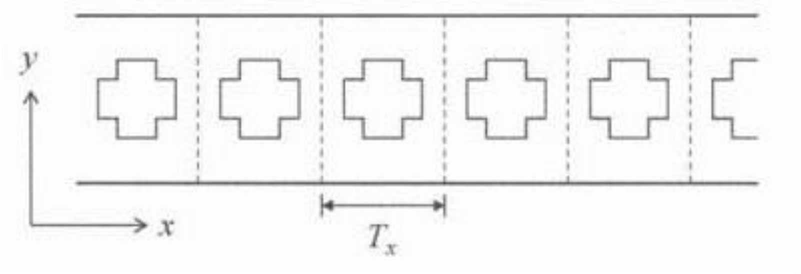

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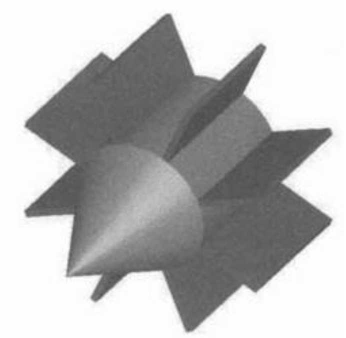

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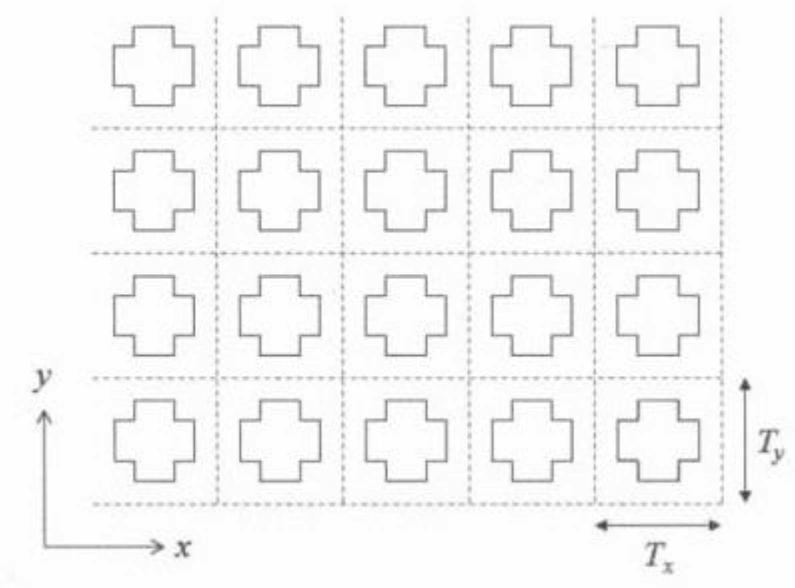

2D Periodic Structure [3]

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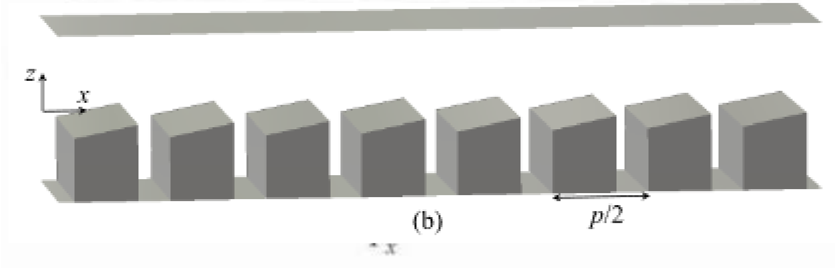

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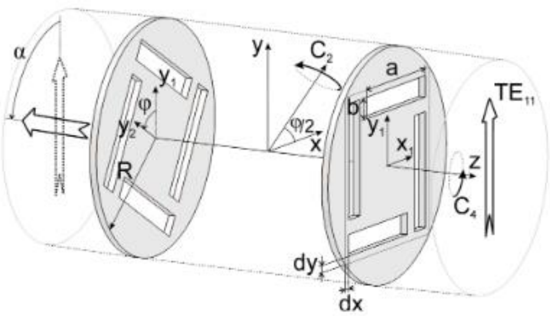

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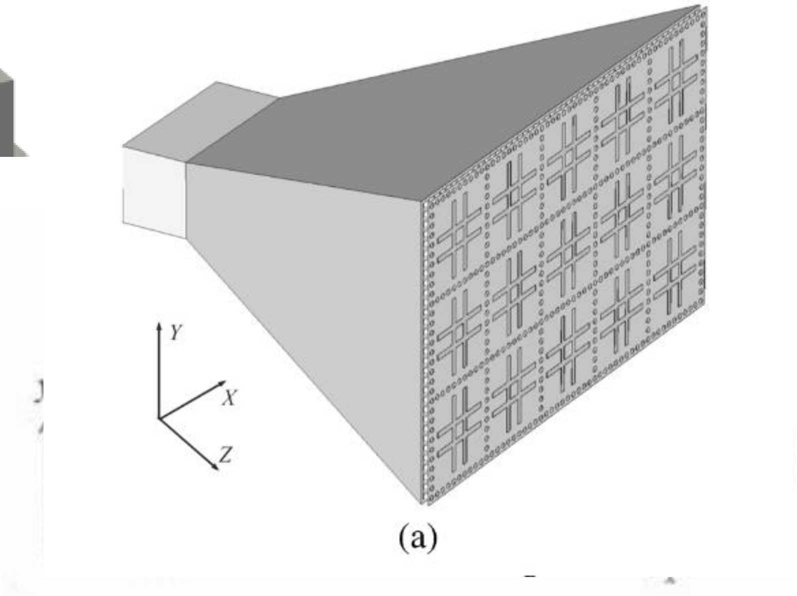

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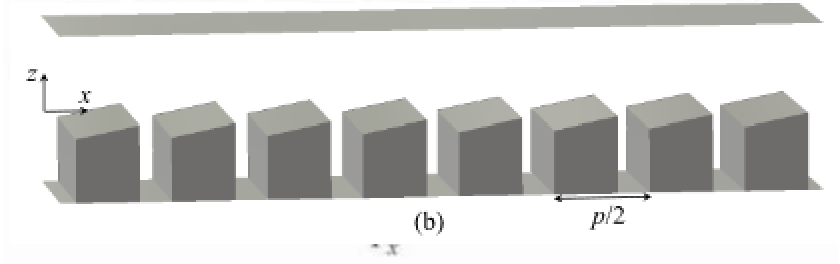

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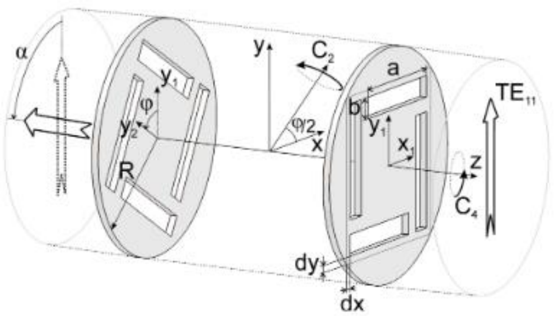

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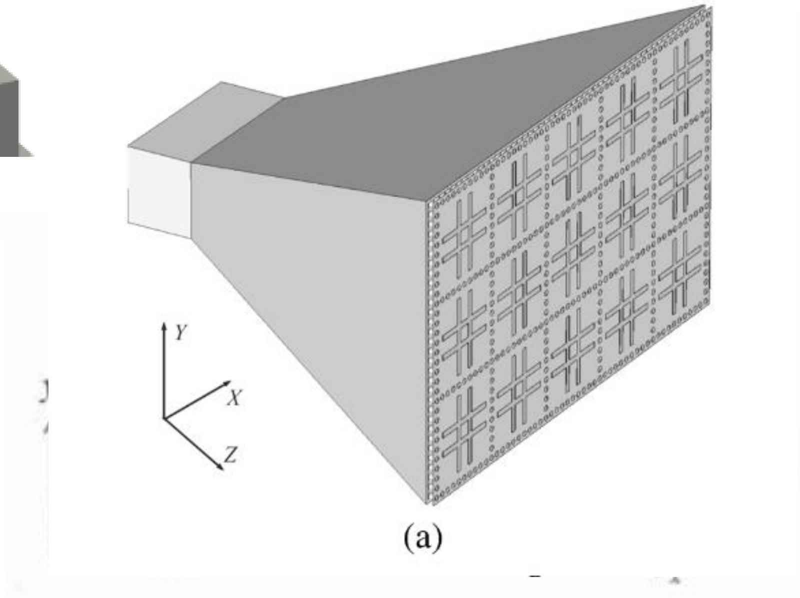

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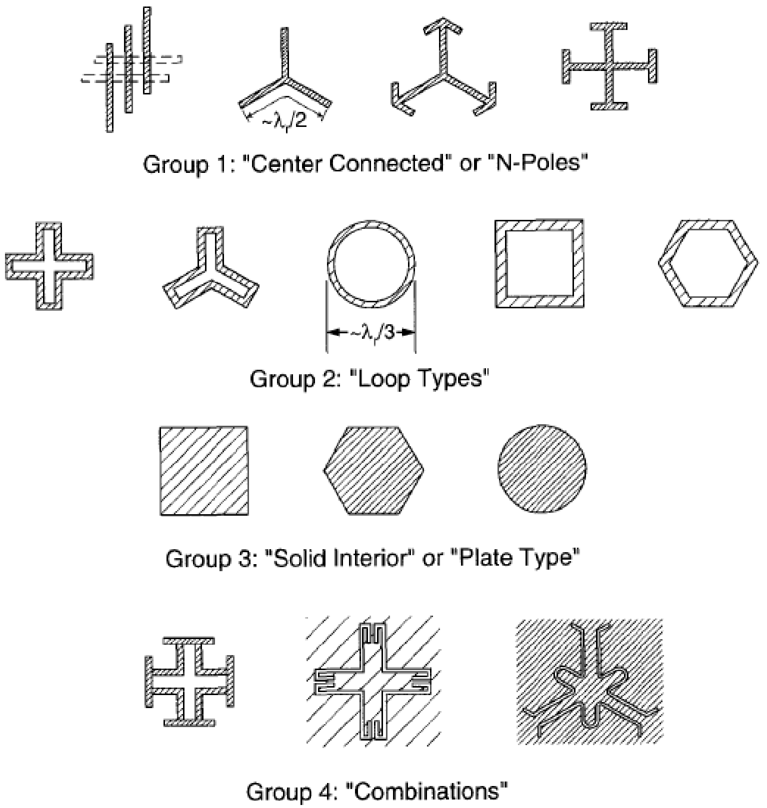

Types of unit cells

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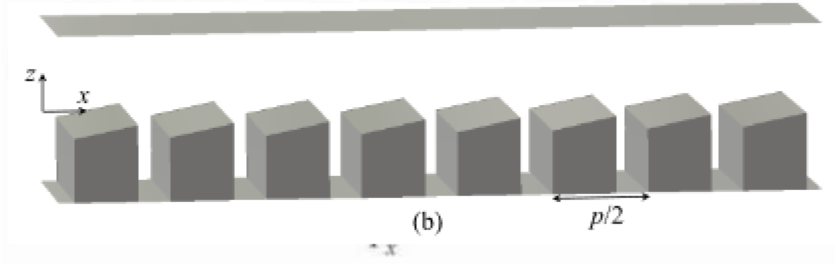

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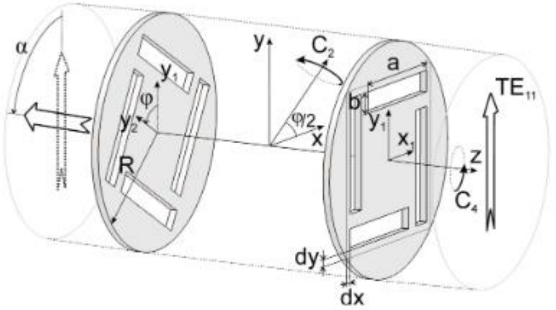

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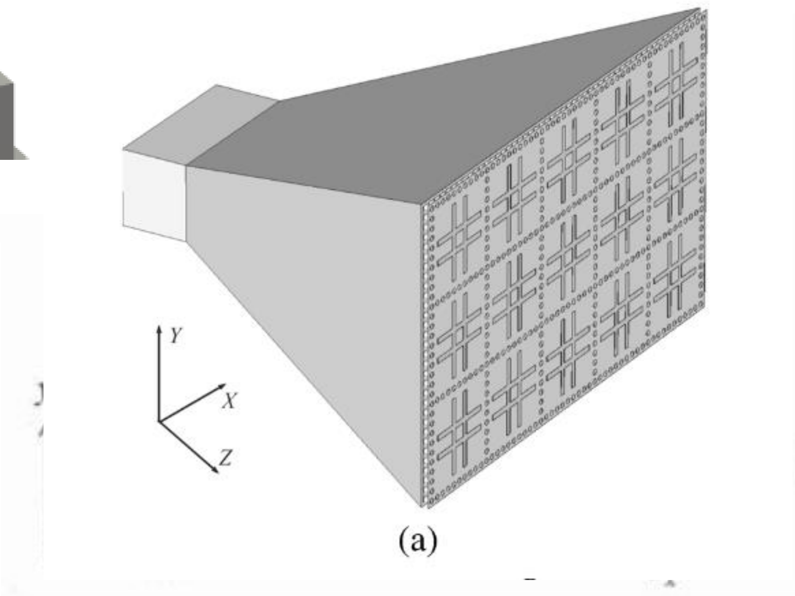

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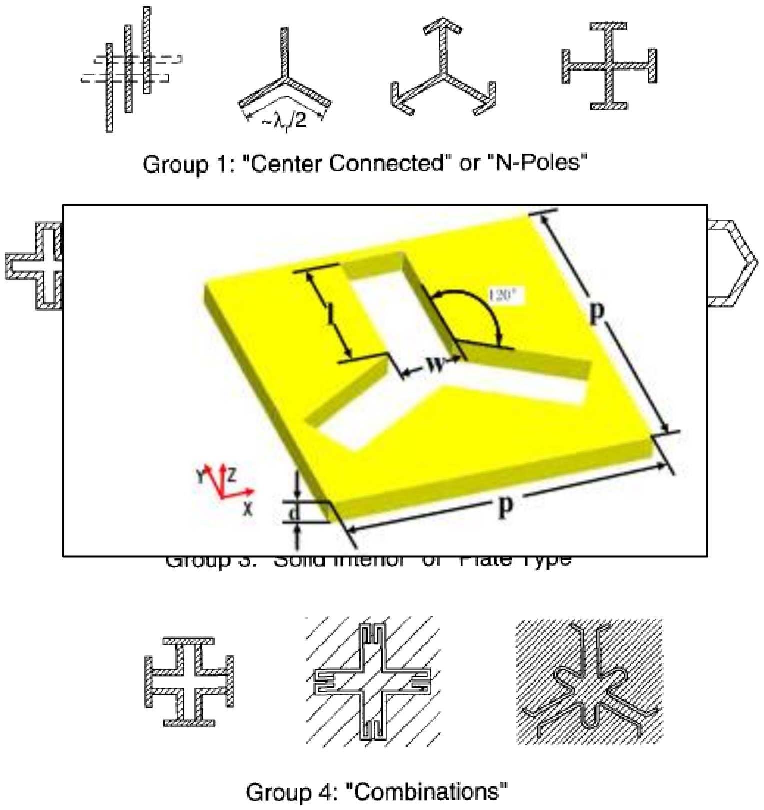

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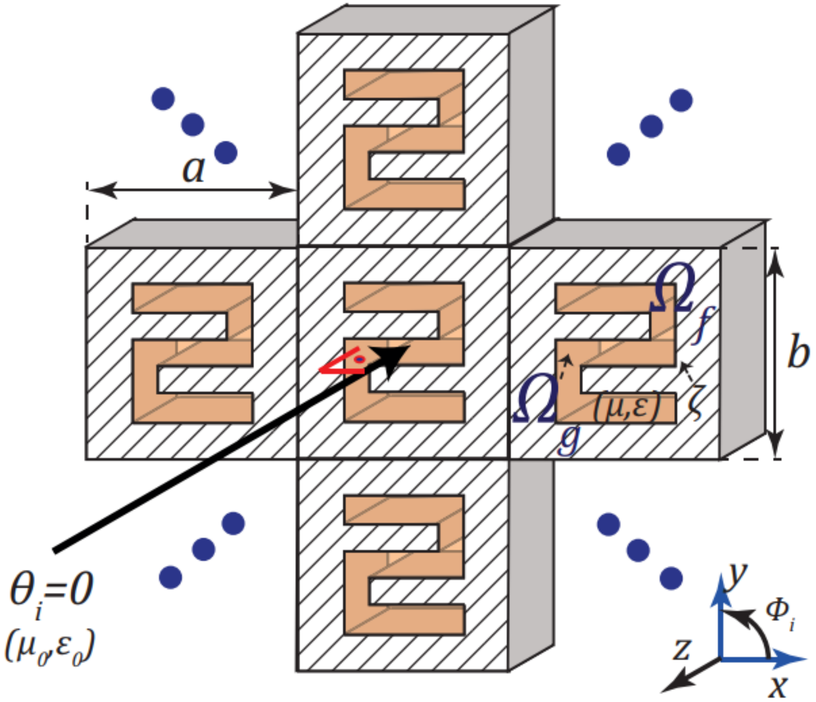
Problem under analysis

- Waveguide-like unit cells under normal incidence
  - Mode-Matching approach



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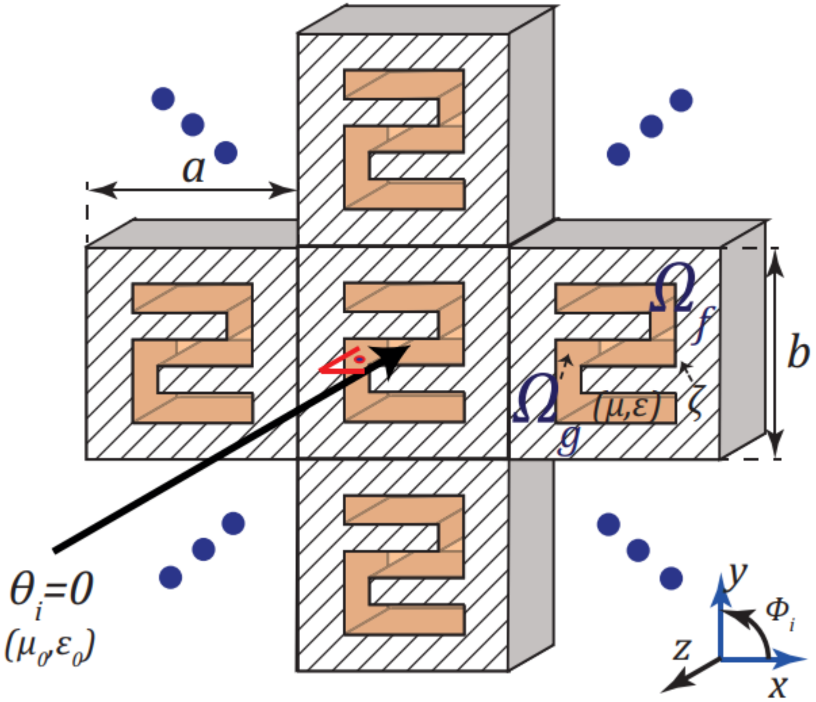

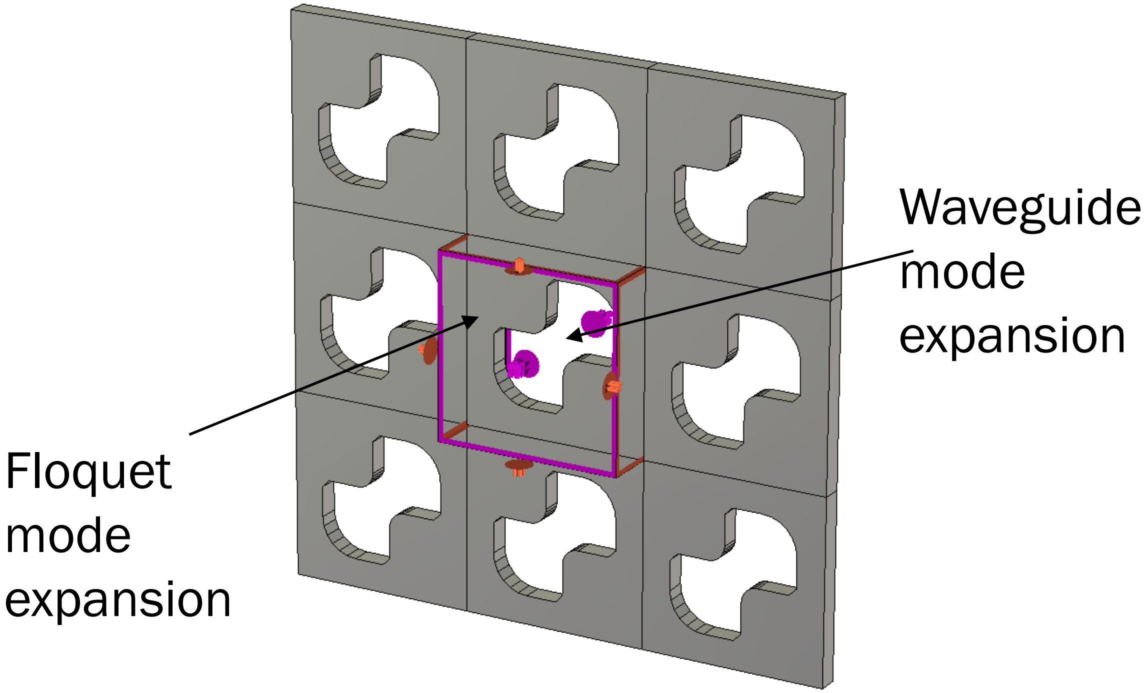

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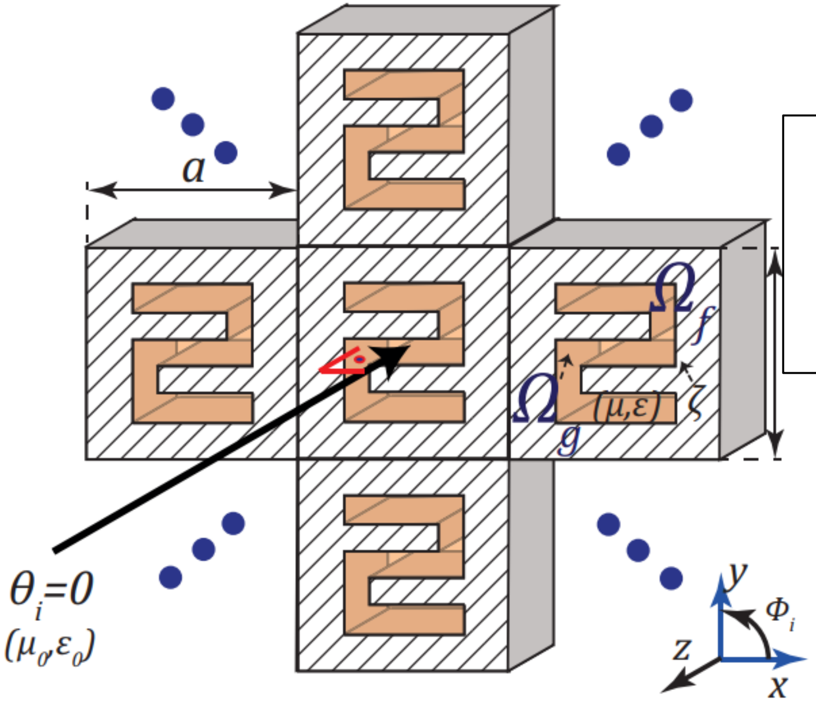
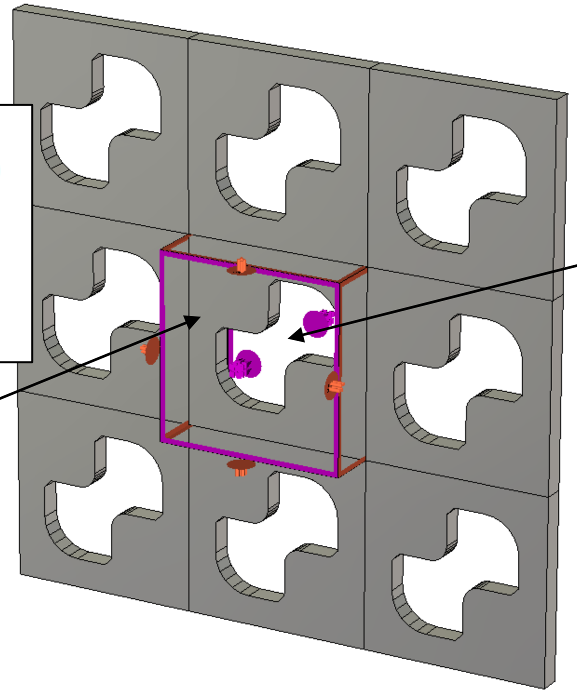


Field continuity equations

$$\boldsymbol{E}|_{\Omega_f,0^-} = \begin{cases} 0, & \text{in } (\Omega_f - \Omega_g) \\ \boldsymbol{E}|_{\Omega_g,0^+}, & \text{in } \Omega_g \end{cases}$$

$$\boldsymbol{H}|_{\Omega_f,0^-} = \boldsymbol{H}|_{\Omega_g,0^+}, \quad \text{in } \Omega_g.$$

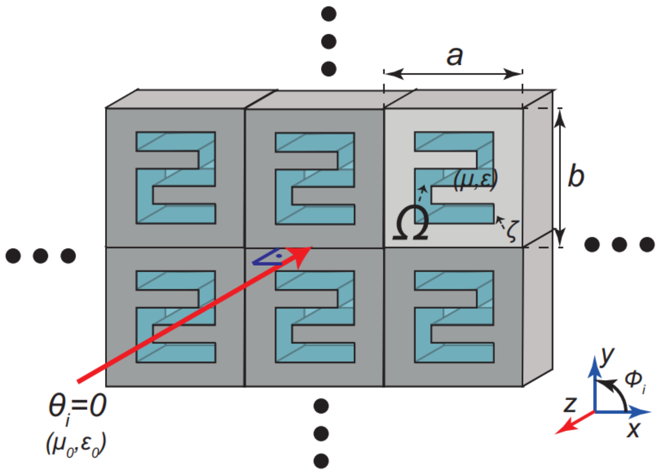


Waveguide mode expansion

Floquet mode expansion

Floquet and Waveguide Modes

- Derived from scalar potentials



$$\boxed{\nabla_t^2\psi + k_c^2\psi = 0}$$

In Ω_f

PBCs

$$\psi_{pq}^{\text{TE, TM}} = \frac{-\jmath}{\sqrt{ab}\sqrt{k_p^2 + k_q^2}} e^{-\jmath(k_p x + k_q y)}$$

$$k_p = k_0 \sin\theta_i \cos\phi_i + \frac{2p\pi}{a}$$

$$k_q = k_0 \sin\theta_i \sin\phi_i + \frac{2q\pi}{b}$$

Floquet and Waveguide Modes

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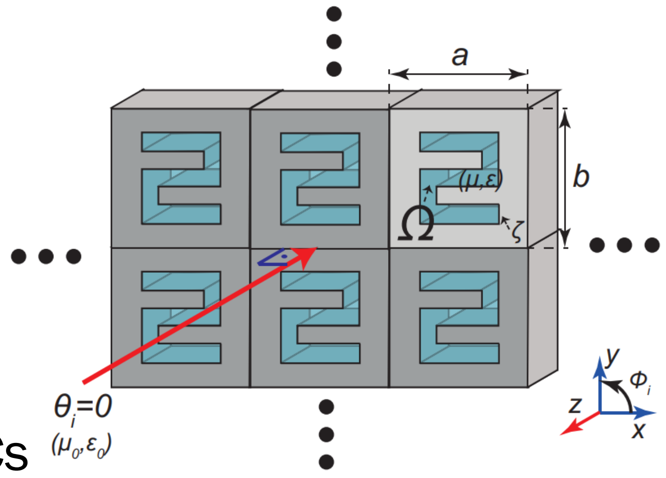

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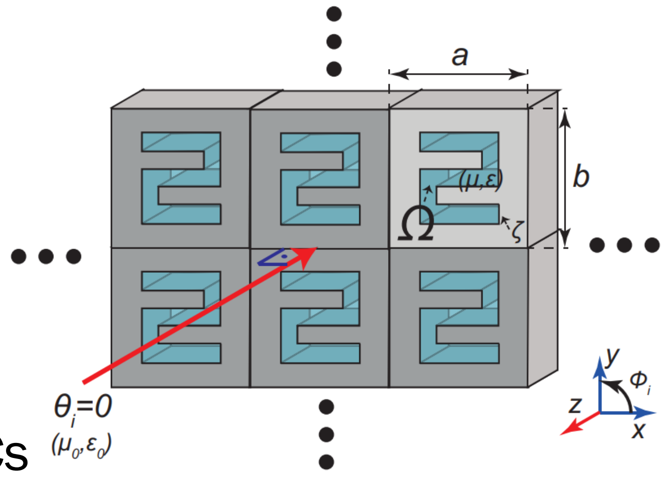
In Ω_g

PEC BCs

- Frequency independent (in this problem)
- Unknown a priori
  - Use a numerical method to solve them (e.g. 2D-FEM)

$$\psi = \sum_k u_k l_k$$

Floquet and Waveguide Modes



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Frequency dependent → consider normal incidence

In Ω_g

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- Unknown a priori
  - Use a numerical method to solve them (e.g. 2D-FEM)

$$\psi = \sum_k u_k l_k$$

TE00

$$\psi_{0_a} = \frac{-x}{\sqrt{ab}}$$

TM00

$$\psi_{0_b} = \frac{-y}{\sqrt{ab}}.$$

Matching on the discontinuity

- Floquet modes do not directly fulfill classic orthogonality relation

$$\langle \boldsymbol{e}_{f,m}, \boldsymbol{h}_{f,n} \rangle_f = \iint_{\Omega_f} \boldsymbol{e}_{f,m} \times \boldsymbol{h}_{f,n} \cdot \hat{\boldsymbol{z}}\, d\Omega \neq A_m \delta_{mn}$$

$$\boxed{\psi_{pq}^{\text{TE, TM}} = \frac{-\jmath}{\sqrt{ab}\sqrt{k_p^2 + k_q^2}} e^{-\jmath(k_p x + k_q y)}}$$

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- Thus, we have decided to use

$$\langle \boldsymbol{e}_{f,m}, \boldsymbol{h}^*_{f,n} \rangle_f = P_m \delta_{mn}.$$

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- In terms of scalar potentials

$$\langle \nabla_t \psi_m, \hat{\boldsymbol{z}} \times \nabla_t \psi_m^* \rangle_{\{f,g\}} = Q_m$$

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- In terms of scalar potentials

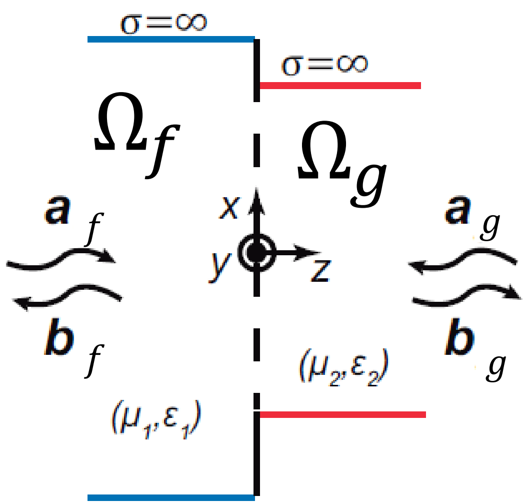
$$\langle \nabla_t \psi_m, \hat{\boldsymbol{z}} \times \nabla_t \psi_m^* \rangle_{\{f,g\}} = 1,$$

From scalar potentials to modal fields

- Modal expansion on both sides of the discontinuity

$$\boldsymbol{E}|_{\Omega_f,0^-} = \sum_{m=-\infty}^{\infty} (a_{f,m} + b_{f,m})\, \boldsymbol{e}_{f,m}$$

$$\boldsymbol{H}|_{\Omega_f,0^-} = \sum_{m=-\infty}^{\infty} (a_{f,m} - b_{f,m})\, \boldsymbol{h}_{f,m}$$



$$\boldsymbol{E}|_{\Omega_g,0^-} = \sum_{j=0}^{\infty} (b_{g,j} + a_{g,j})\, \boldsymbol{e}_{g,i}$$

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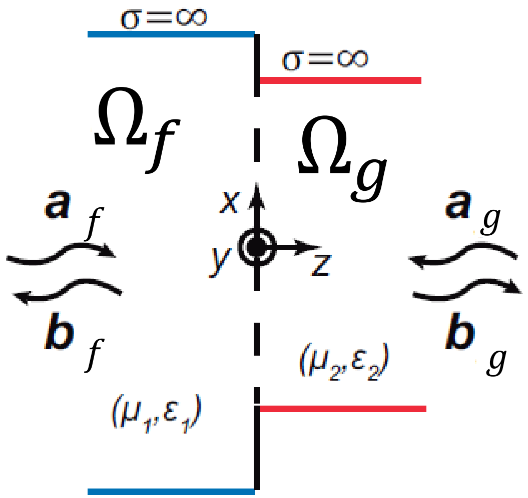

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$$\boldsymbol{H}|_{\Omega_g,0^-} = \sum_{j=0}^{\infty} (b_{g,j} - a_{g,j})\, \boldsymbol{h}_{g,j}$$

$$\boldsymbol{e}_m^{\mathrm{TE}} = Q_m^{\frac{1}{2}\mathrm{TE}} Z_m^{\frac{1}{2}\mathrm{TE}} (\nabla_t \psi_m^{\mathrm{TE}} \times \hat{\boldsymbol{z}}) \quad \boldsymbol{h}_m^{\mathrm{TE}} = Q_m^{\frac{1}{2}\mathrm{TE}} Y_m^{\frac{1}{2}\mathrm{TE}} \nabla_t \psi_m^{\mathrm{TE}}$$

$$\boldsymbol{e}_m^{\mathrm{TM}} = Q_m^{\frac{1}{2}\mathrm{TM}} Z_m^{\frac{1}{2}\mathrm{TM}} \nabla_t \psi_m^{\mathrm{TM}} \quad \boldsymbol{h}_m^{\mathrm{TM}} = Q_m^{\frac{1}{2}\mathrm{TM}} Y_m^{\frac{1}{2}\mathrm{TM}} (\hat{\boldsymbol{z}} \times \nabla_t \psi_m^{\mathrm{TM}})$$

From scalar potentials to modal fields

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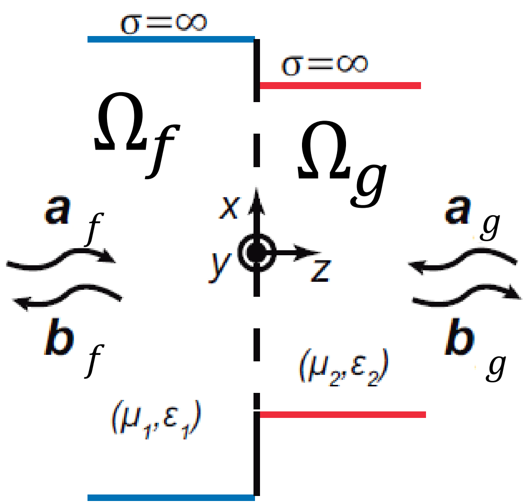

$$\boldsymbol{E}|_{\Omega_g,0^-} = \sum_{j=0}^{\infty} (b_{g,j} + a_{g,j})\, \boldsymbol{e}_{g,i}$$

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$$\boldsymbol{e}_m^{\mathrm{TE}} = Q_m^{\frac{1}{2}\mathrm{TE}} \boxed{Z_m^{\frac{1}{2}\mathrm{TE}}} (\nabla_t \psi_m^{\mathrm{TE}} \times \hat{\boldsymbol{z}}) \qquad \boldsymbol{h}_m^{\mathrm{TE}} = Q_m^{\frac{1}{2}\mathrm{TE}} \boxed{Y_m^{\frac{1}{2}\mathrm{TE}}} \nabla_t \psi_m^{\mathrm{TE}}$$

$$\boldsymbol{e}_m^{\mathrm{TM}} = Q_m^{\frac{1}{2}\mathrm{TM}} \boxed{Z_m^{\frac{1}{2}\mathrm{TM}}} \nabla_t \psi_m^{\mathrm{TM}} \qquad \boldsymbol{h}_m^{\mathrm{TM}} = Q_m^{\frac{1}{2}\mathrm{TM}} \boxed{Y_m^{\frac{1}{2}\mathrm{TM}}} (\hat{\boldsymbol{z}} \times \nabla_t \psi_m^{\mathrm{TM}})$$

Only frequency dependent terms under normal incidence

Matching on the discontinuity

- Final system of equations

$$(\boldsymbol{a}_f + \boldsymbol{b}_f) = \boldsymbol{X}_c\,(\boldsymbol{b}_g + \boldsymbol{a}_g)$$
$$\boldsymbol{X}\,(\boldsymbol{a}_f - \boldsymbol{b}_f) = (\boldsymbol{b}_g - \boldsymbol{a}_g)$$

$$\boldsymbol{X} = \boldsymbol{Z}_f^{1/2}\bar{\mathbf{X}}\mathbf{Y}_g^{1/2} \quad \boldsymbol{X}_c = (\boldsymbol{Z}_f^{1/2}\bar{\mathbf{X}}^*\mathbf{Y}_g^{1/2})^T$$

- Generalized scattering matrix

$$\boldsymbol{S}_{\mathrm{GSM}} = \begin{bmatrix} \boldsymbol{X}_c\boldsymbol{F}\boldsymbol{X} - \boldsymbol{I}_f & \boldsymbol{X}_c\boldsymbol{F} \\ \boldsymbol{F}\boldsymbol{X} & \boldsymbol{F} - \boldsymbol{I}_g \end{bmatrix}$$

$$\boldsymbol{F} = 2(\boldsymbol{I}_f + \boldsymbol{X}\boldsymbol{X}_c)^{-1}$$

Matching on the discontinuity

- Final system of equations

$$
\begin{aligned}
(\boldsymbol{a}_f + \boldsymbol{b}_f) &= \boldsymbol{X}_c\,(\boldsymbol{b}_g + \boldsymbol{a}_g) \\
\boldsymbol{X}\,(\boldsymbol{a}_f - \boldsymbol{b}_f) &= (\boldsymbol{b}_g - \boldsymbol{a}_g)
\end{aligned}
$$

$$
\boldsymbol{X} = \boldsymbol{Z}_f^{1/2}\boxed{\bar{\boldsymbol{X}}}\boldsymbol{Y}_g^{1/2} \quad \boldsymbol{X}_c = (\boldsymbol{Z}_f^{1/2}\boxed{\bar{\boldsymbol{X}}^*}\boldsymbol{Y}_g^{1/2})^T
$$

Due to using the complex conjugate to keep orthogonality

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Due to using the complex conjugate to keep orthogonality

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$$\boldsymbol{F} = 2(\boldsymbol{I}_f + \boldsymbol{X}\boldsymbol{X}_c)^{-1}$$

- Normalized cross-product matrix

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Completely frequency independent

2D-FEM Hybridization

- Floquet modes

$$\boldsymbol{T}\boldsymbol{u}_{f,m} = \boldsymbol{d}_m,$$

  – Project to 2D-FEM function space

$$d_m(i) = \frac{-\jmath}{\sqrt{k_p^2 + k_q^2}\sqrt{ab}} \iint_{\Omega_g} e^{-\jmath(k_p x + k_q y)} l_i \, d\Omega.$$

FEM matrices

$$R(i,j) = \iint_{\Omega_g} \nabla_t l_i \times \nabla_t l_j \cdot \hat{\boldsymbol{z}} \, d\Omega$$

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$$\boldsymbol{S}_{\mathrm{GSM}} = \begin{bmatrix} \boldsymbol{X}_c \boldsymbol{F} \boldsymbol{X} - \boldsymbol{I}_f & \boldsymbol{X}_c \boldsymbol{F} \\ \boldsymbol{F} \boldsymbol{X} & \boldsymbol{F} - \boldsymbol{I}_g \end{bmatrix} \quad \boldsymbol{F} = 2(\boldsymbol{I}_f + \boldsymbol{X} \boldsymbol{X}_c)^{-1}$$

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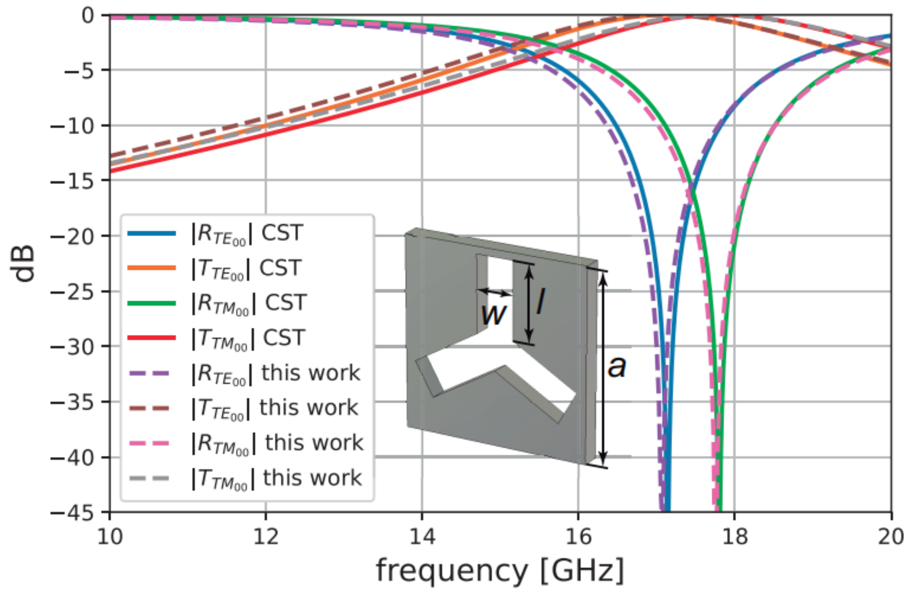
Also works for oblique incidence, but all matrices must be recomputed for each freq.

Results

- Validation results

"Thick", Y-shaped FSS



- Results compared against CST
- The formulation works well on both thick and thin FSSs

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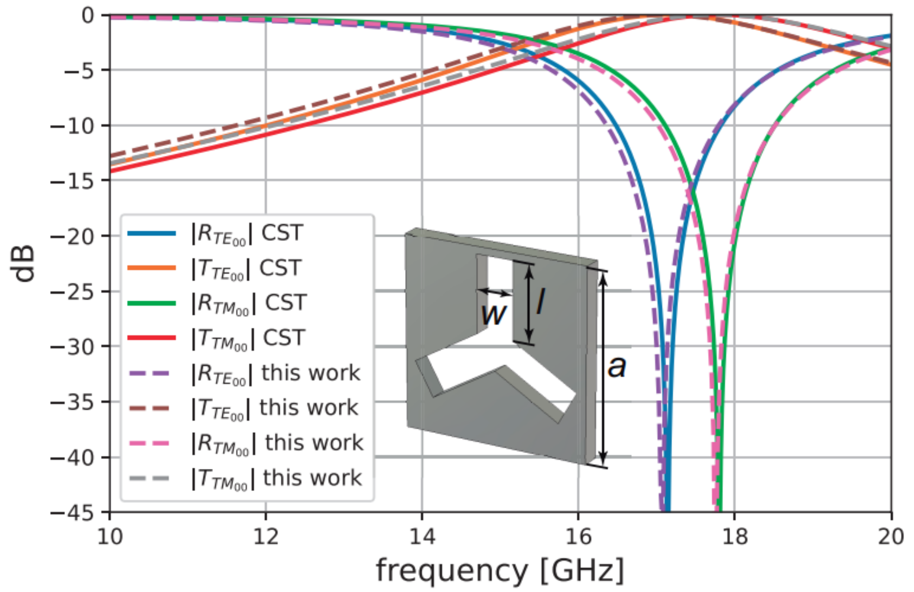
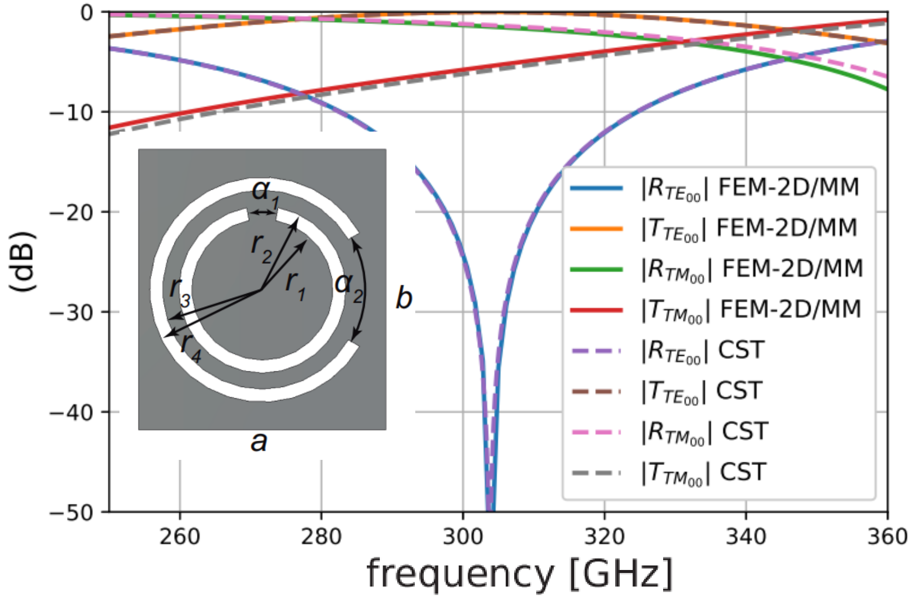


Polarization converter



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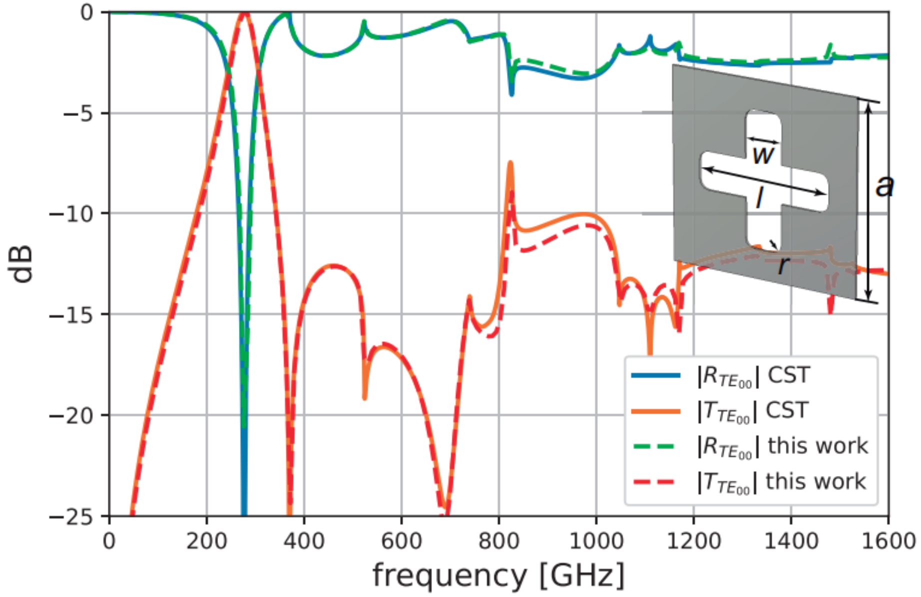
- Convergence & Performance

Rounded-Edge cross unit cell



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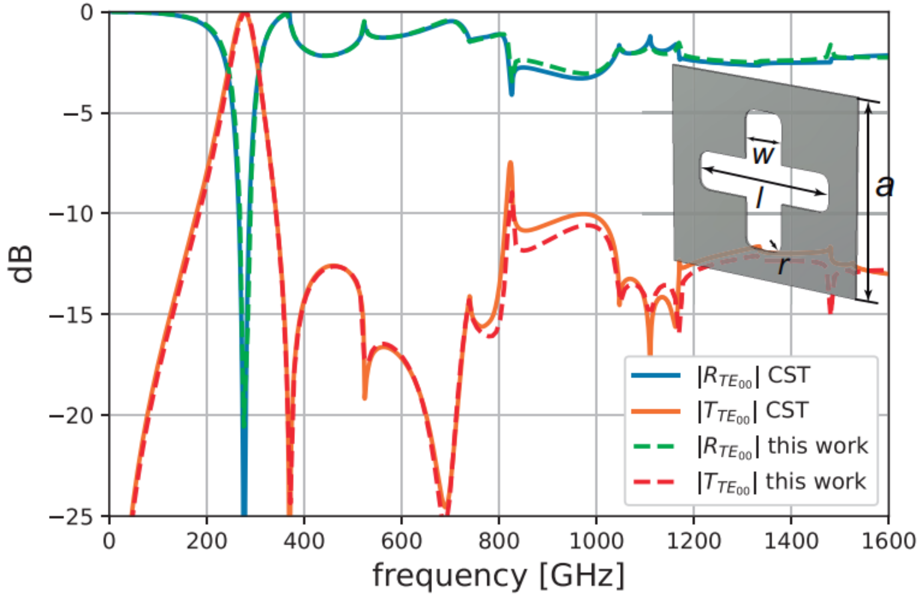
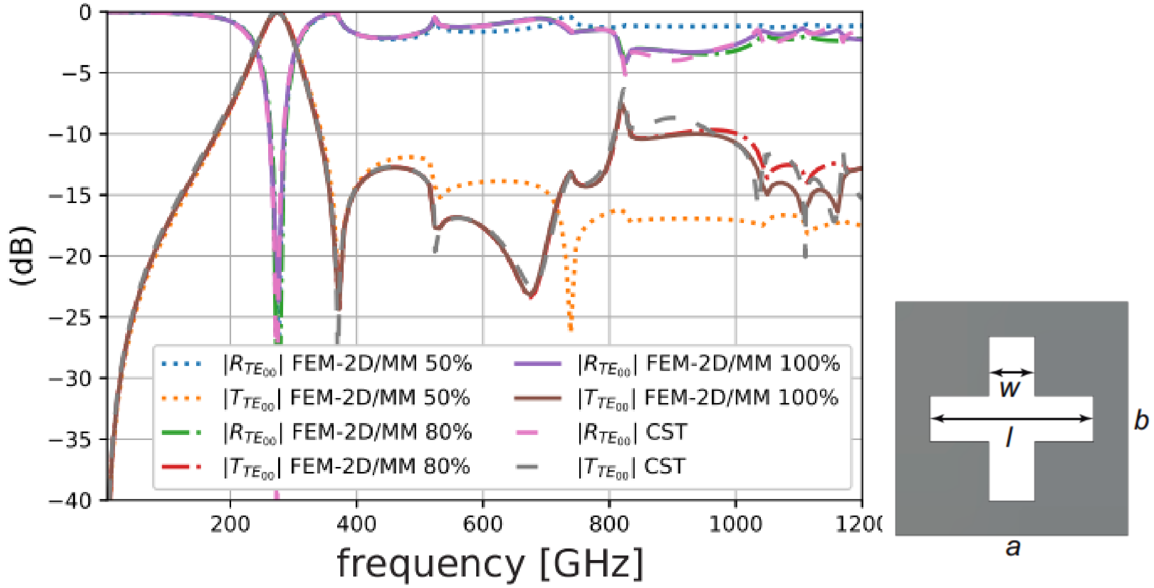


Convergence on cross unit cell



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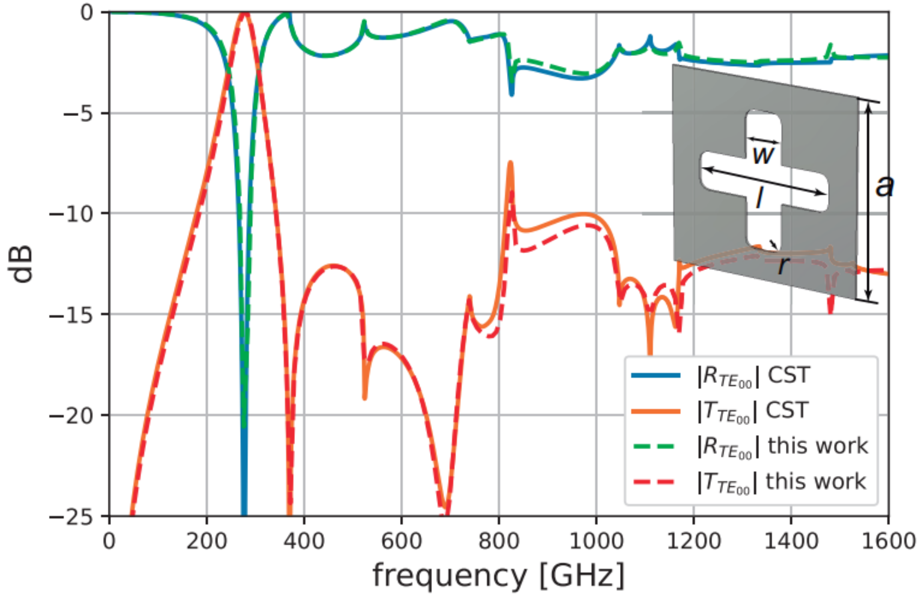

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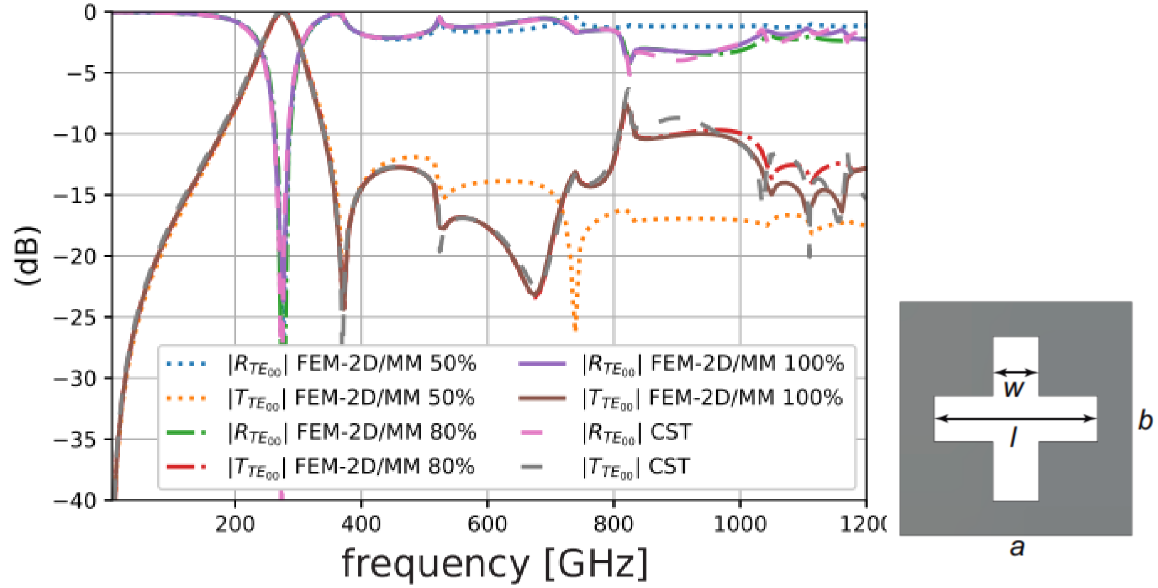

Rounded-edge cross	# modes	Comp. time	Time per freq. point
CST	122 Floquet	>30 min	~30 s
2D-FEM/MM	400 Floquet, 32 Wg	~10 s	~0.01 s

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- Hybridization of 2D-FEM/MM for the waveguide-like FSS problem.
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Future work

- Extend the formulation to other unit cells
- Extend the formulation beyond normal incidence

We3E-5

Normal Incidence Scattering of Waveguide-Like FSS/PSS in Scalar 2D-FEM/MM Extracting the Frequency dependence

G. Garcia-Contreras, J. Córcoles, J. A. Ruiz-Cruz
Universidad Autónoma de Madrid, Madrid, Spain