

We3F-1

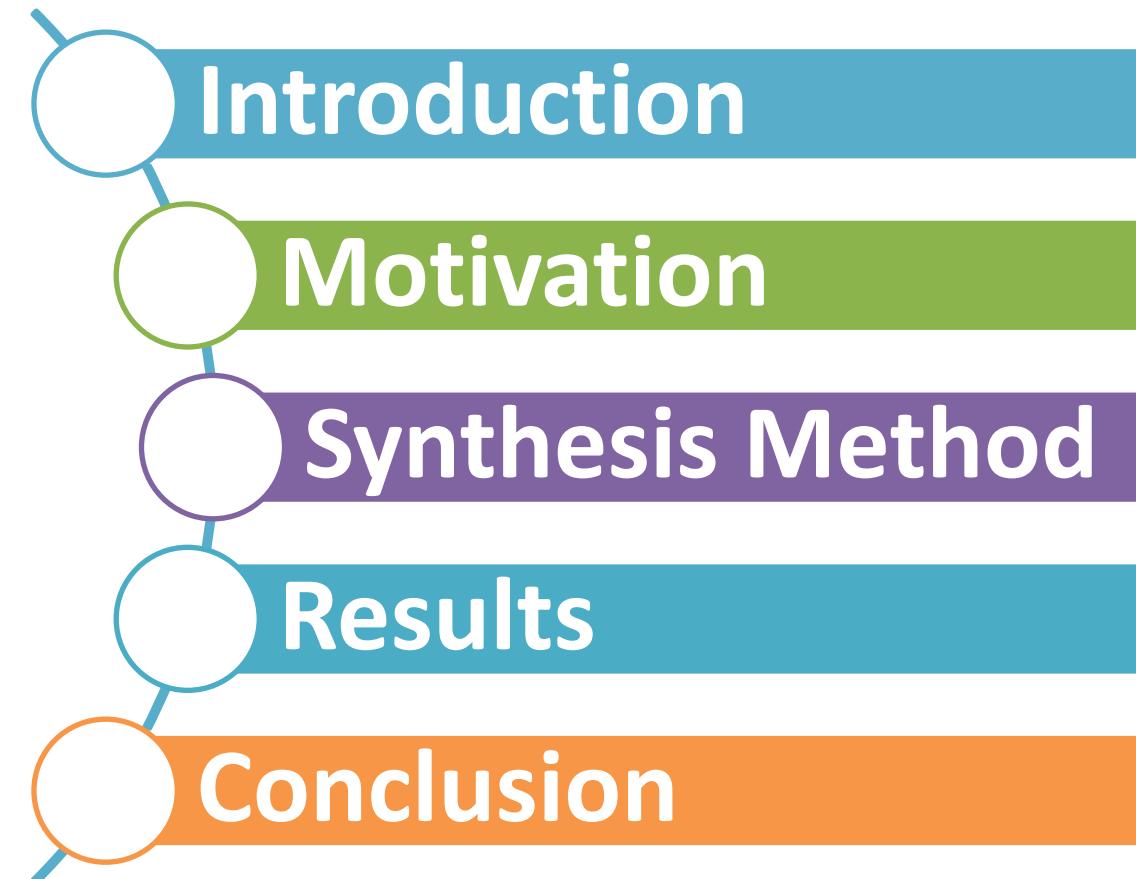
# Synthesis of Wideband Cross-Coupled Resonator Filter for Direct Circuit Implementation Using Lumped Elements

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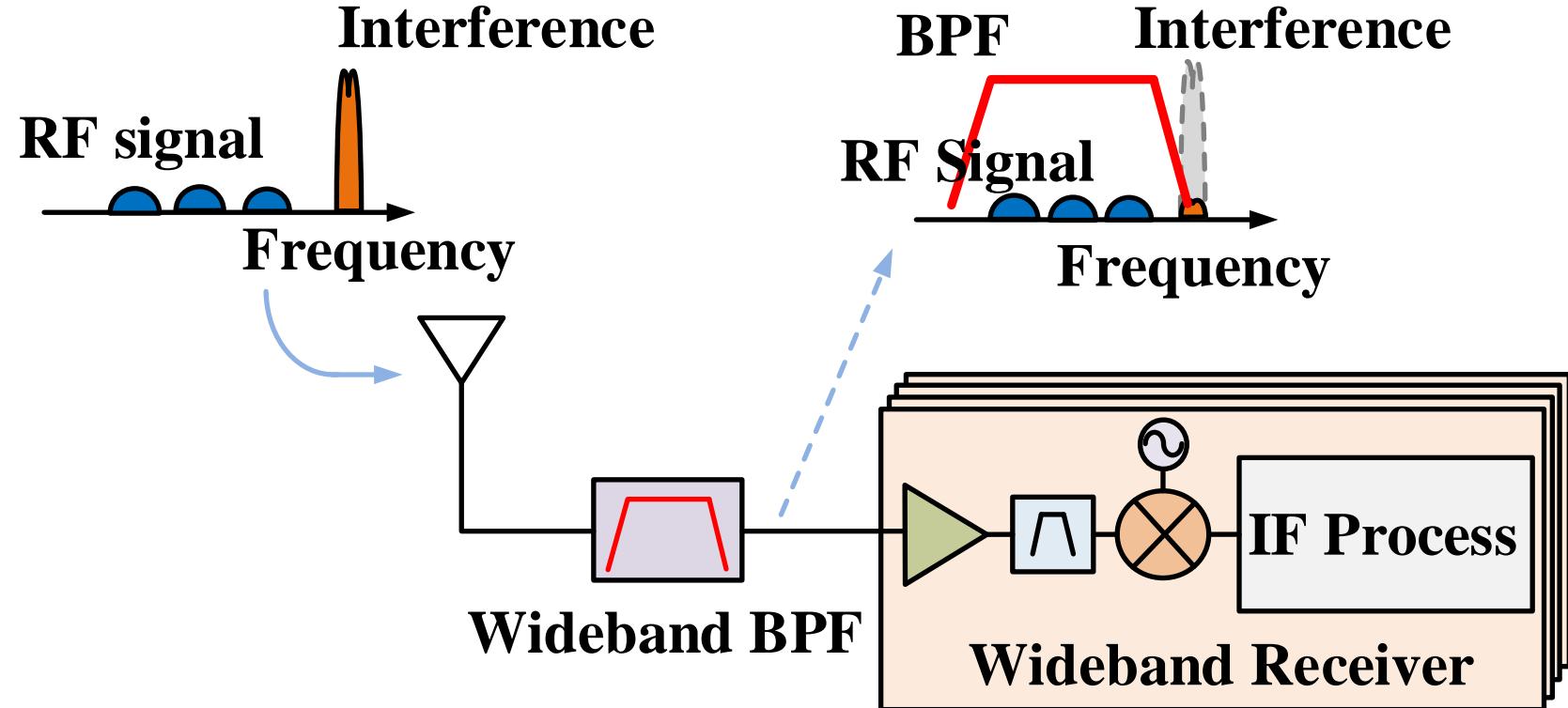
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# Outline



# Introduction

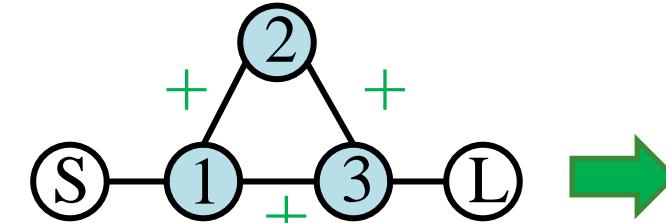
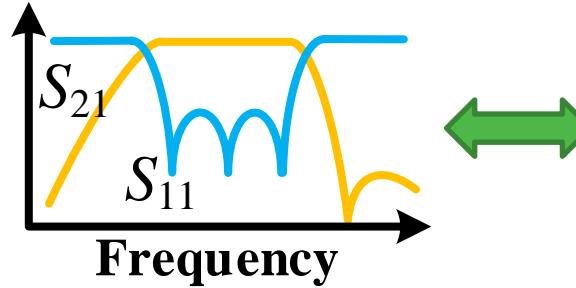


**Wideband filter in wideband receiver system**

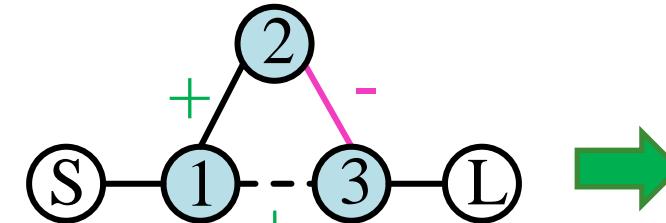
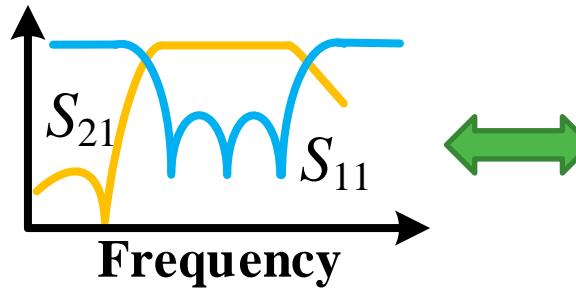
# Introduction

Ref.	Matrix elements	Circuit implementation
Cameron's [12]	$\omega C + B, B$	Narrowband; Approximation at $\omega_0$ 
Amari's [3]	$\omega C - \frac{1}{\omega L}, \pm \frac{1}{\omega L}$	Wideband; <b>Negative inductance</b> 
Wei Meng's [4]	$\omega C - \frac{1}{\omega L}, \frac{1}{\omega L}$ or $-\omega C$	Wideband; <b>Only Inline</b> 
F. J. Chen [18]	$\omega C + B, \omega C + B$	Moderate bandwidth 
This work	$\omega C - \frac{1}{\omega L}, \frac{1}{\omega L}$ and $-\omega C$	Wideband; Circuit implement 

# Motivation

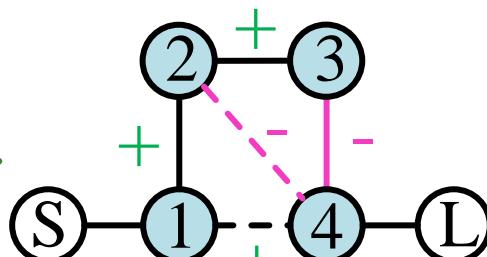
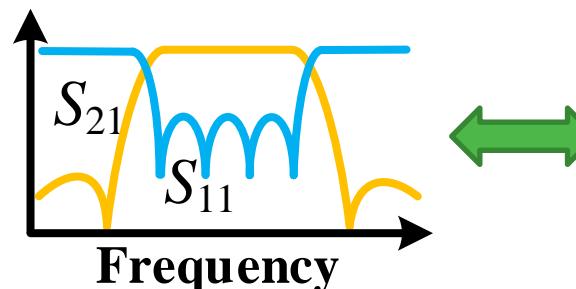


$$\frac{1}{\omega L}, \frac{1}{\omega L}$$



$$\frac{1}{\omega L}, -\frac{1}{\omega L}$$

$$\boxed{\frac{1}{\omega L}, -\omega C}$$



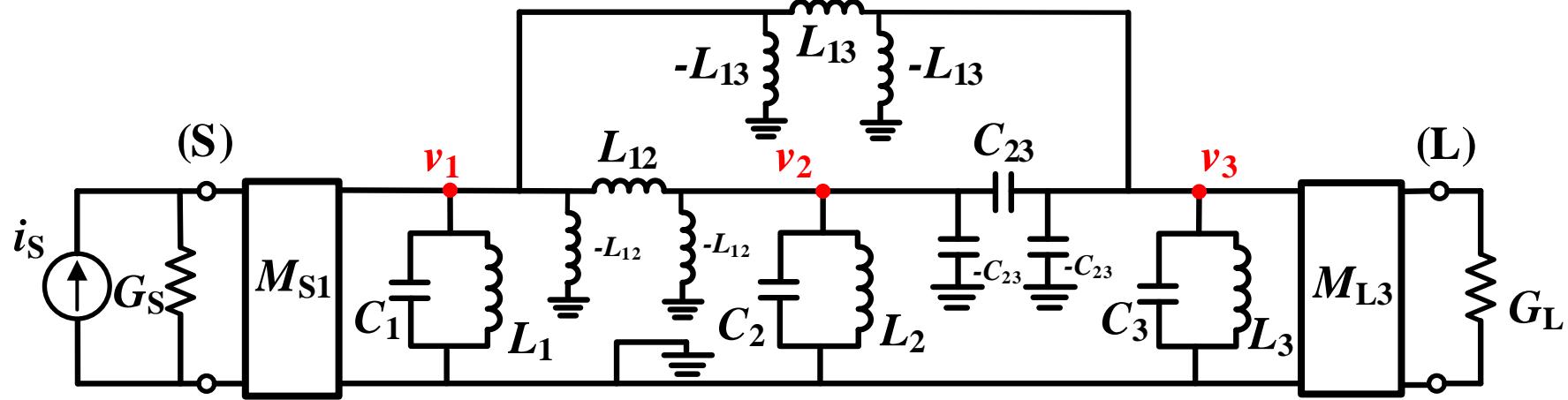
$$\frac{1}{\omega L}, -\frac{1}{\omega L}$$

$$\boxed{\frac{1}{\omega L}, -\omega C}$$



Transform the **negative inductance** to **positive capacitance**

# Motivation



$$\begin{bmatrix}
 G_S & jM_{S1} & 0 & 0 & 0 \\
 jM_{S1} & j\omega C_1 + \frac{\omega_1^2}{j\omega} & -\frac{1}{j\omega L_{12}} & -\frac{1}{j\omega L_{13}} & 0 \\
 0 & -\frac{1}{j\omega L_{12}} & j\omega C_2 + \frac{\omega_2^2}{j\omega} & -j\omega C_{23} & 0 \\
 0 & -\frac{1}{j\omega L_{13}} & -j\omega C_{23} & j\omega C_3 + \frac{\omega_3^2}{j\omega} & jM_{3L} \\
 0 & 0 & 0 & jM_{3L} & G_L
 \end{bmatrix}
 \begin{bmatrix}
 v_S \\
 v_1 \\
 v_2 \\
 v_3 \\
 v_L
 \end{bmatrix}
 = \begin{bmatrix}
 i_S \\
 0 \\
 0 \\
 0 \\
 0
 \end{bmatrix}$$

$$[Y] \cdot [v] = [i]$$



$$S_{21} = 2\sqrt{G_S G_L} [Y]_{(N+2),1}^{-1}$$

# Motivation

$$A = \begin{bmatrix} -j & M_{S1} & 0 & 0 & 0 \\ M_{S1} & \omega C_1 - \frac{1}{\omega L_1} & \frac{1}{\omega L_{12}} & \frac{1}{\omega L_{13}} & 0 \\ 0 & \frac{1}{\omega L_{12}} & \omega C_2 - \frac{1}{\omega L_2} & -\omega C_{23} & 0 \\ 0 & \frac{1}{\omega L_{13}} & -\omega C_{23} & \omega C_3 - \frac{1}{\omega L_3} & M_{3L} \\ 0 & 0 & 0 & M_{3L} & -j \end{bmatrix}$$

$$[Y] = j[A]$$

Admittance matrix

$$M = \frac{1}{\omega L};$$

$$M = -\omega C$$

Represent

$J$  inverters

$$J = \frac{1}{\omega L};$$

$$J = \omega C$$

Implement by

$$S_{21} = -j2[A]_{(N+2),1}^{-1}$$

$$A_1 = \begin{bmatrix} s & 1 & 2 & 3 & L \\ -j & M'_{s1} & 0 & 0 & 0 \\ M'_{s1} & \omega - \frac{\omega_1'^2}{\omega} & \frac{1}{\omega L'_{12}} & -\frac{1}{\omega L'_{13}} & 0 \\ 0 & \frac{1}{\omega L'_{12}} & \omega - \frac{\omega_2'^2}{\omega} & \frac{1}{\omega L'_{23}} & 0 \\ 0 & -\frac{1}{\omega L'_{13}} & \frac{1}{\omega L'_{23}} & \omega - \frac{\omega_3'^2}{\omega} & M'_{3L} \\ 0 & 0 & 0 & M'_{3L} & -j \end{bmatrix}$$

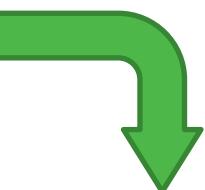
**Matrix similarity Transformation**

$$S = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \\ L \end{bmatrix}$$

$$A_2 = T A_1 T^\top$$

Orthogonal matrix:

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & Pm & 0 \\ 0 & 0 & 0 & c_1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$



$$A_2 =$$

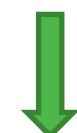
$$\begin{bmatrix} -j & M'_{s1} & 0 \\ M'_{s1} & \omega - \frac{\omega'_1{}^2}{\omega} & \frac{1}{\omega L'_{12}} \\ 0 & \frac{1}{\omega L'_{12}} & \omega - \frac{\omega'_2{}^2}{\omega} \\ 0 & \left( \frac{Pm}{L'_{12}} - \frac{c_1}{L'_{13}} \right) \frac{1}{\omega} & Pm \cdot \omega + \left( \frac{c_1}{L'_{23}} - Pm \cdot \omega'_2{}^2 \right) \frac{1}{\omega} \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & & & & 0 \\ \left( \frac{Pm}{L'_{12}} - \frac{c_1}{L'_{13}} \right) \frac{1}{\omega} & & & & 0 \\ Pm \cdot \omega + \left( \frac{c_1}{L'_{23}} - Pm \cdot \omega'_2{}^2 \right) \frac{1}{\omega} & & & & 0 \\ \left( Pm^2 + c_1{}^2 \right) \omega + \left( \frac{c_1}{L'_{23}} Pm - Pm^2 \omega'_2{}^2 - c_1{}^2 \omega'_3{}^2 + \frac{Pm \cdot c_1}{L'_{23}} \right) \frac{1}{\omega} & & & & c_1 M'_{3L} \\ c_1 M'_{3L} & & & & -j \end{bmatrix}$$

Conditions:

$$-Pm \cdot \omega'_2{}^2 + c_1 / L'_{23} = 0$$

$$Pm / L'_{12} - c_1 / L'_{13} > 0$$



Solve equations:

$$Pm = \frac{c_1}{\omega'_2{}^2 L'_{23}}, \quad \left( \frac{1}{\omega'_2{}^2 L'_{12} L'_{23}} - \frac{1}{L'_{13}} \right) \cdot c_1 < 0$$

$$A_2 =$$

$$\begin{bmatrix} -j & M'_{S1} & 0 & 0 & 0 \\ M'_{S1} & \omega - \frac{\omega'^2}{\omega} & \frac{1}{\omega L'_{12}} & \left( \frac{c_1}{\omega'^2 L'^2_{12}} - \frac{c_1}{L'_{13}} \right) \frac{1}{\omega} & 0 \\ 0 & \frac{1}{\omega L'_{12}} & \omega - \frac{\omega'^2}{\omega} & \frac{c_1}{\omega'^2 L'_{12}} \omega & 0 \\ 0 & \left( \frac{c_1}{\omega'^2 L'^2_{12}} - \frac{c_1}{L'_{13}} \right) \frac{1}{\omega} & \frac{c_1}{\omega'^2 L'_{12}} \omega & M'_{33} & c_1 \cdot M'_{3L} \\ 0 & 0 & 0 & c_1 \cdot M'_{3L} & -j \end{bmatrix}$$

$$M'_{33} = \left( \frac{c_1^2}{\omega'^4 L'^2_{12}} + c_1^2 \right) \omega + c_1^2 \left( \frac{1}{\omega'^2 L'^2_{12}} - \omega'^2_3 \right) \frac{1}{\omega}$$

$$T_s = \text{diag}(1, 1/M'_{S1}, \sqrt{C_2}, |1/(c_1 M'_{3L})|, 1)$$



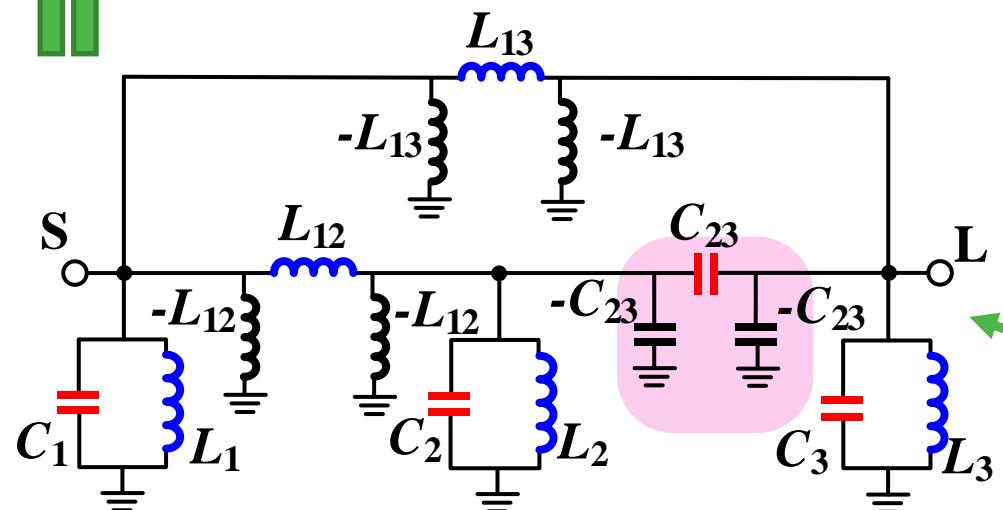
$$A_3 = T_s A_2 T_s^\top$$

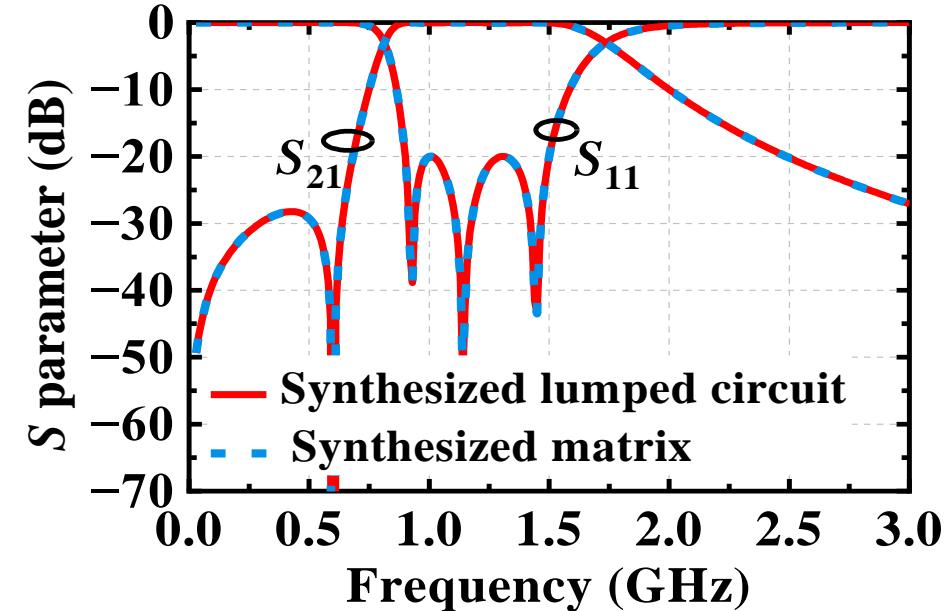
$$A_3 = \begin{bmatrix} -j & 1 & 0 & 0 & 0 \\ 1 & \omega C_1 - \frac{1}{\omega L_1} & \frac{1}{\omega L_{12}} & \frac{1}{\omega L_{13}} & 0 \\ 0 & \frac{1}{\omega L_{12}} & \omega C_2 - \frac{1}{\omega L_2} & -\omega C_{23} & 0 \\ 0 & \frac{1}{\omega L_{13}} & -\omega C_{23} & \omega C_3 - \frac{1}{\omega L_3} & -1 \\ 0 & 0 & 0 & -1 & -j \end{bmatrix}$$

# Third-order synthesis example

**Matrix**

$$A_3 = \begin{bmatrix} -j & 1 & 0 & 0 & 0 \\ 1 & \omega C_1 - \frac{1}{\omega L_1} & \frac{1}{\omega L_{12}} & \frac{1}{\omega L_{13}} & 0 \\ 0 & \frac{1}{\omega L_{12}} & \omega C_2 - \frac{1}{\omega L_2} & -\omega C_{23} & 0 \\ 0 & \frac{1}{\omega L_{13}} & -\omega C_{23} & \omega C_3 - \frac{1}{\omega L_3} & -1 \\ 0 & 0 & 0 & -1 & -j \end{bmatrix}$$

**Circuit**

**Specifications:**

 Passband: 0.9-1.5 GHz;  
 TZ: 0.6 GHz; RL: 20 dB

**Circuit parameters:**
 $C_1 = 226.3 \text{ pF}, C_2 = 250 \text{ pF}, C_3 = 295.4 \text{ pF},$   
 $L_1 = 0.063 \text{ nH}, L_2 = 0.084 \text{ nH}, L_3 = 0.08 \text{ nH},$   
 $L_{12} = 0.16 \text{ nH}, C_{23} = 131 \text{ pF}, L_{13} = 0.72 \text{ nH}.$

$$B_1 = \begin{bmatrix} -j & M'_{s1} & 0 & 0 & 0 & 0 \\ M'_{s1} & \omega - \frac{\omega'^2}{\omega} & \frac{1}{\omega L'_{12}} & 0 & -\frac{1}{\omega L'_{14}} & 0 \\ 0 & \frac{1}{\omega L'_{12}} & \omega - \frac{\omega'^2}{\omega} & \frac{1}{\omega L'_{23}} & -\frac{1}{\omega L'_{24}} & 0 \\ 0 & 0 & \frac{1}{\omega L'_{23}} & \omega - \frac{\omega'^2}{\omega} & \frac{1}{\omega L'_{34}} & 0 \\ 0 & -\frac{1}{\omega L'_{14}} & -\frac{1}{\omega L'_{24}} & \frac{1}{\omega L'_{34}} & \omega - \frac{\omega'^2}{\omega} & M'_{4L} \\ 0 & 0 & 0 & 0 & M'_{4L} & -j \end{bmatrix}$$

Orthogonal matrix:

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & Pg & 0 \\ 0 & 0 & 0 & 1 & Pn & 0 \\ 0 & 0 & 0 & 0 & c_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Matrix similarity Transformation

$$B_2 = TB_1 T^\top$$



$B_2 =$ 

$$\begin{bmatrix} -j & M'_{s1} & 0 & 0 \\ * & \omega - \frac{\omega_1'^2}{\omega} & \frac{1}{\omega L'_{12}} & 0 \\ 0 & * & \omega - \frac{\omega_2'^2}{\omega} & \frac{1}{\omega L'_{23}} \\ 0 & 0 & * & \omega - \frac{\omega_3'^2}{\omega} \\ 0 & * & * & * \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned} & Pg \cdot \omega + \left( \frac{Pg}{L'_{12}} - \frac{c_1}{L'_{14}} \right) \frac{1}{\omega} \\ & Pg \cdot \omega + \left( -Pg \cdot \omega_2'^2 + \frac{Pn}{L'_{23}} - \frac{c_1}{L'_{24}} \right) \frac{1}{\omega} \\ & Pg \cdot \omega + \left( -Pn \cdot \omega_3'^2 + \frac{Pg}{L'_{23}} + \frac{c_1}{L'_{34}} \right) \frac{1}{\omega} \\ & k_4 \omega - \frac{\omega_4^2}{\omega} \end{aligned}$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ M'_{4L} \\ -j \end{bmatrix}$$

**Conditions:**

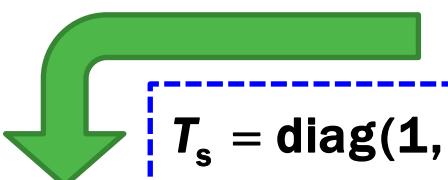
$$\begin{aligned} -Pg \cdot \omega_2'^2 + \frac{Pn}{L'_{23}} - \frac{c_1}{L'_{24}} &= 0 \\ -Pn \cdot \omega_3'^2 + \frac{Pg}{L'_{23}} + \frac{c_1}{L'_{34}} &= 0 \\ \frac{Pg}{L'_{12}} - \frac{c_1}{L'_{14}} &> 0 \end{aligned}$$

**Solve equations:**

 $Pg, Pn$ 

$B_3 = T_s B_2 T_s^\top$

$$T_s = \text{diag}(1, 1/M'_{s1}, \sqrt{C_2}, \sqrt{C_3}, |1/M'_{4L}|, 1)$$



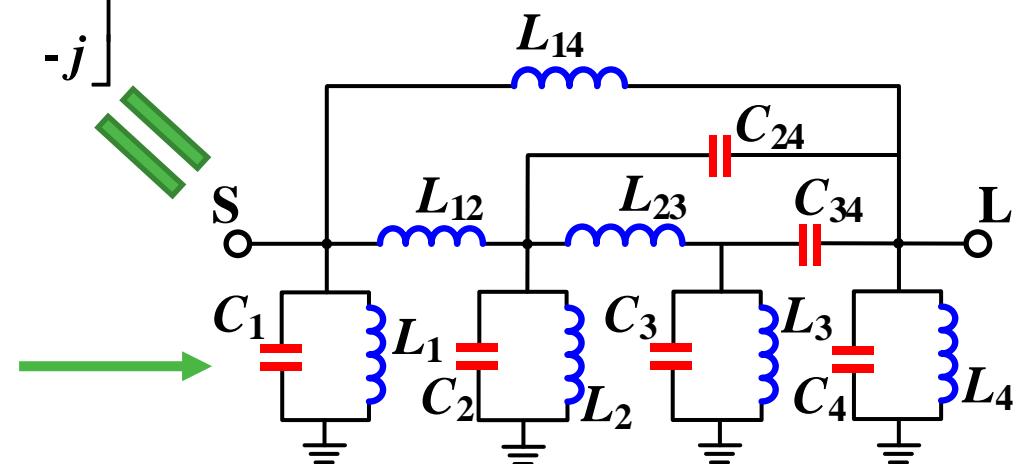
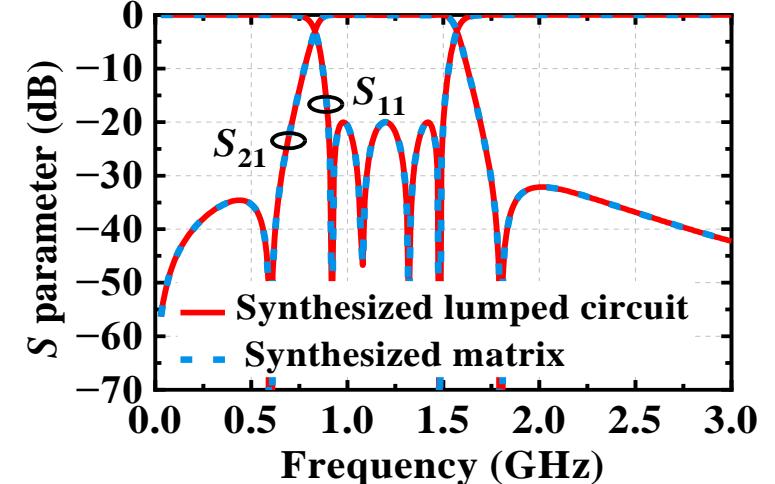
# Fourth-order synthesis example

$$B_3 = \begin{bmatrix} -j & 1 & 0 & 0 & 0 \\ 1 & \omega C_1 - \frac{1}{\omega L_1} & \frac{1}{\omega L_{12}} & 0 & \frac{1}{\omega L_{14}} \\ 0 & \frac{1}{\omega L_{12}} & \omega C_2 - \frac{1}{\omega L_2} & \frac{1}{\omega L_{23}} & -\omega C_{24} \\ 0 & 0 & \frac{1}{\omega L_{23}} & \omega C_3 - \frac{1}{\omega L_3} & -\omega C_{34} \\ 0 & \frac{1}{\omega L_{14}} & -\omega C_{24} & -\omega C_{34} & \omega C_4 - \frac{1}{\omega L_4} \\ 0 & 0 & 0 & 0 & -j \end{bmatrix}$$

### Circuit parameters:

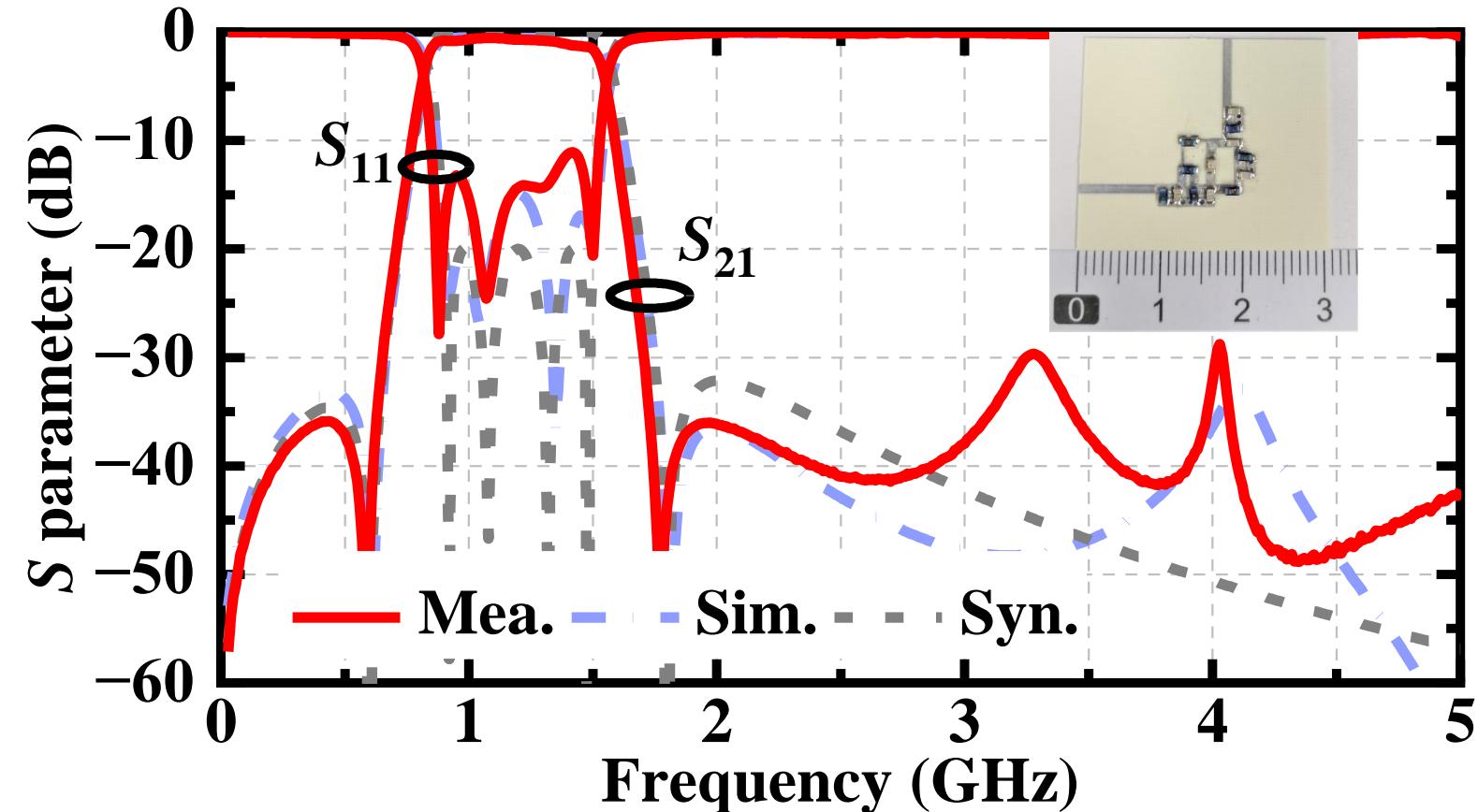
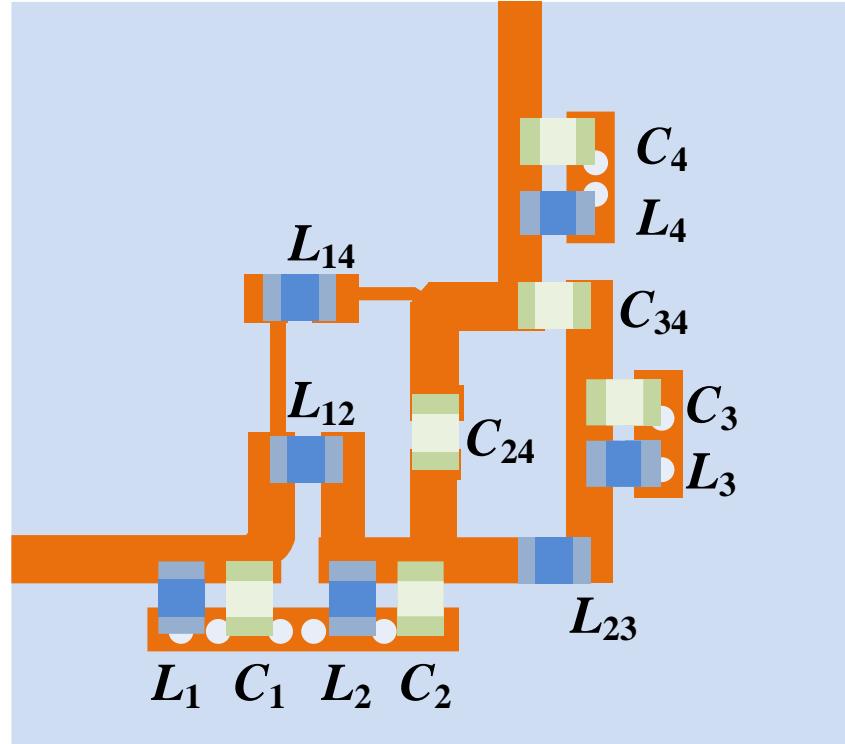
$C_1=5 \text{ pF}$ ,  $C_2=4.5 \text{ pF}$ ,  $C_3=2.68 \text{ pF}$ ,  $C_4=3.4 \text{ pF}$ ,  $L_1=5.49 \text{ nH}$ ,  
 $L_2=11 \text{ nH}$ ,  $L_3=5.49 \text{ nH}$ ,  $L_4=3.96 \text{ nH}$ ,  $L_{12}=7.95 \text{ nH}$ ,  
 $L_{23}=9.26 \text{ nH}$ ,  $C_{34}=2.3 \text{ pF}$ ,  $L_{14}=69.8 \text{ nH}$ ,  $C_{24}=0.46 \text{ pF}$ .

Passband: 0.9-1.5 GHz  
 TZ: 0.6 GHz; RL: 20 dB



# Measurement

Inductor Capacitor



# Conclusion

- To synthesize wideband cross coupled resonator filter;
- All **negative inductive-coupling** elements can be transformed to **positive capacitive** coupling elements
  
- can fully represent the **frequency-dependent characteristics** of the *J*-inverters and resonators
- Can be **directly implemented using lumped element**

# THANKS

Questions and comments are welcome!