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Quantum Models for Flux-Driven Superconducting Traveling-Wave Parametric Amplifiers with Different Nonlinear Junction Topologies

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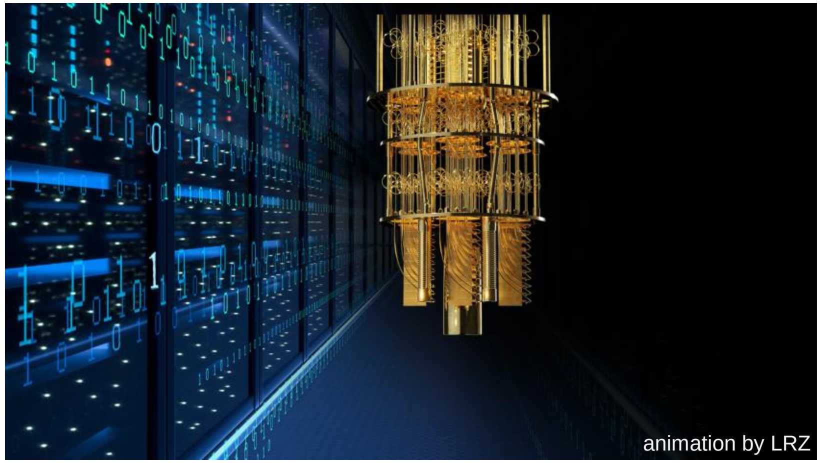
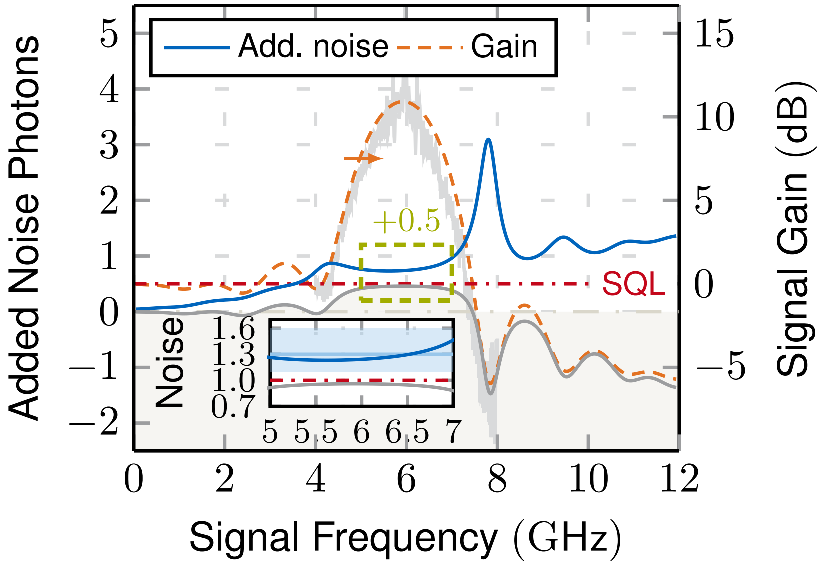
Problem

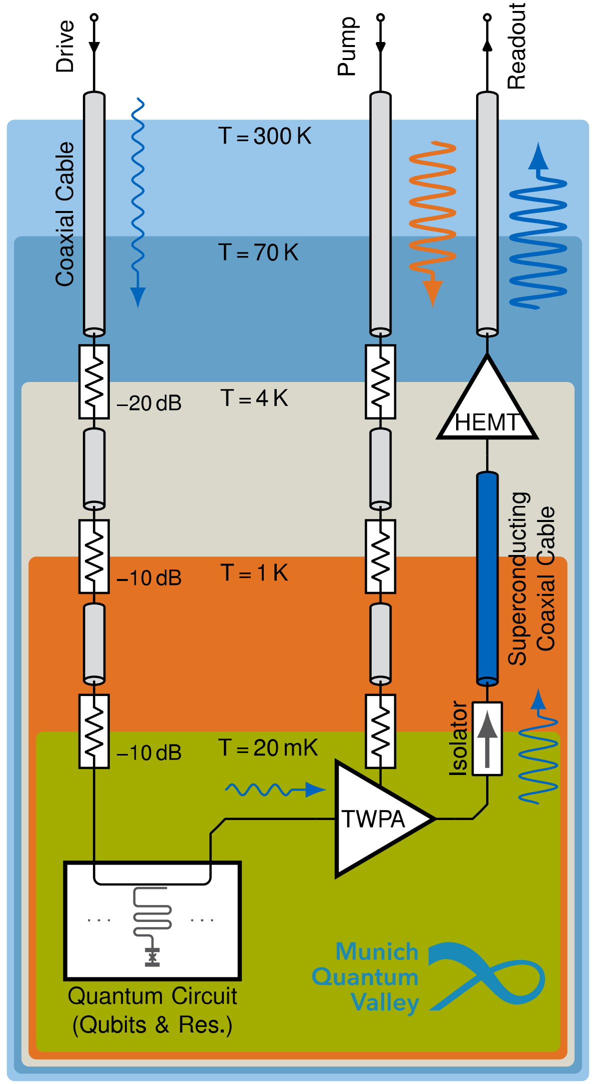




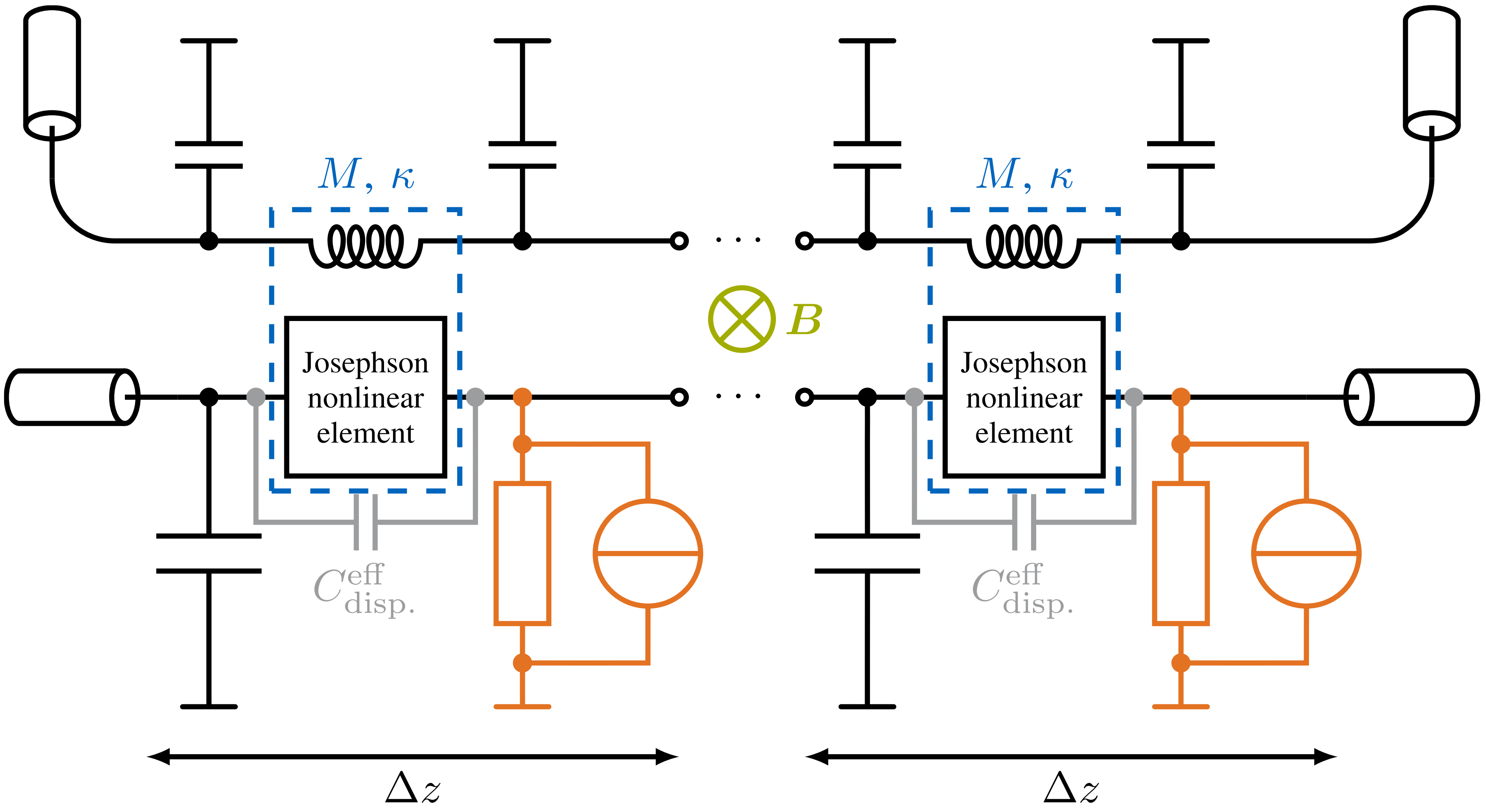
Yuan *et al.*, Phys. Rev. A **107** 022612 (2023)

- How to probe a qubit?
- Dispersive readout schemes
- Ultra-low-power signals
- High readout fidelity requires good SNR!

- Added noise needs to be close to the quantum limit!



Flux-Driven TWPAs – Circuit Model



Flux-Driven TWPAs – Taylor Expansion

- Taylor expansion of the current around a flux point φ^* with $i(\varphi^*) = 0$

$$i(\varphi) = i'(\varphi^*, \varphi_{\mathrm{ext}}) \cdot (\varphi - \varphi^*) + \mathcal{O}(\varphi^2)\,.$$

- Parametric amplification is achieved by a time-varying modulation of the linear inductance

- The effective potential with the effective flux operator

$$\hat{\varphi}_{\mathrm{eff}} = \hat{\varphi} - \varphi^*$$

is given by

$$U_{\mathrm{eff}} = \varphi_0 \int_{\varphi^*}^{\varphi} i(\varphi')\,\mathrm{d}\varphi' = \frac{\varphi_0}{2} i'(\varphi^*, \varphi_{\mathrm{ext}}) \cdot \varphi_{\mathrm{eff}}^2\,.$$

Taylor expansion around Bias Flux

$$\begin{aligned}\hat{U} = {} & \frac{\varphi_0}{2} i'(\varphi^*, \varphi_{\mathrm{ext}})\Big|_{\varphi_{\mathrm{ext}}=\varphi_{\mathrm{B}}} \hat{\varphi}_{\mathrm{eff}}^2 \\ & + \frac{\varphi_0}{2} \frac{\mathrm{d}i'(\varphi^*, \varphi_{\mathrm{ext}})}{\mathrm{d}\varphi_{\mathrm{ext}}}\Big|_{\varphi_{\mathrm{ext}}=\varphi_{\mathrm{B}}} \underbrace{(\varphi_{\mathrm{ext}} - \varphi_{\mathrm{B}})}_{\varphi_{\mathrm{ac}}} \hat{\varphi}_{\mathrm{eff}}^2 \\ & + \frac{\varphi_0}{4} \frac{\mathrm{d}^2 i'(\varphi^*, \varphi_{\mathrm{ext}})}{\mathrm{d}\varphi_{\mathrm{ext}}{}^2}\Big|_{\varphi_{\mathrm{ext}}=\varphi_{\mathrm{B}}} \underbrace{(\varphi_{\mathrm{ext}} - \varphi_{\mathrm{B}})^2}_{\varphi_{\mathrm{ac}}^2} \hat{\varphi}_{\mathrm{eff}}^2\,.\end{aligned}$$

Nonlinear Coefficients

- By factoring out the critical current I_c, we can identify the nonlinear coefficients

$$c_2 = \frac{1}{I_\mathrm{c}} i'(\varphi^*, \varphi_\mathrm{ext})\Big|_{\varphi_\mathrm{ext}=\varphi_\mathrm{B}},$$

Linear inductance

$$c_3 = \frac{1}{2I_\mathrm{c}} \frac{\mathrm{d}i'(\varphi^*, \varphi_\mathrm{ext})}{\mathrm{d}\varphi_\mathrm{ext}}\Big|_{\varphi_\mathrm{ext}=\varphi_\mathrm{B}},$$

Three-wave mixing

$$c_4 = \frac{1}{4I_\mathrm{c}} \frac{\mathrm{d}^2 i'(\varphi^*, \varphi_\mathrm{ext})}{\mathrm{d}{\varphi_\mathrm{ext}}^2}\Big|_{\varphi_\mathrm{ext}=\varphi_\mathrm{B}}.$$

Cross-phase modulation

- The phase operator can be expressed in terms of creation and annihilation operators

$$\hat{\varphi} = \frac{\Delta z}{\sqrt{2\pi}} \int_0^\infty \frac{k(\omega)}{\varphi_0 \omega} \sqrt{\frac{\hbar k(\omega)}{2C'}} \hat{a}_\omega \mathrm{e}^{\mathrm{i}k(\omega)z - \mathrm{i}\omega t} \, \mathrm{d}\omega + \mathrm{H.c.}$$

Three-Wave Mixing Hamiltonian

- Resulting Three-wave mixing Hamiltonian

$$\hat{H}_{\mathrm{WM}} = -\int_0^x\int_0^\infty\int_0^\infty \mathrm{d}\omega\,\mathrm{d}\omega'\,\mathrm{d}z\,\frac{\mathrm{i}\hbar\kappa\omega_{\mathrm{p}}C' f_\Lambda}{16\pi I_{\mathrm{c}}}\cdot\frac{c_3}{c_2^2}\times$$
$$\times\left[A_{\mathrm{p},0}\hat{a}^\dagger_\omega\hat{a}^\dagger_{\omega'}\mathrm{e}^{\mathrm{i}[k_{\mathrm{p}}-k(\omega)-k(\omega')]z-\mathrm{i}(\omega_{\mathrm{p}}-\omega-\omega')t}+\mathrm{H.c.}\right]$$
$$+\int_0^x\int_0^\infty\int_0^\infty \mathrm{d}\omega\,\mathrm{d}\omega'\,\mathrm{d}z\,\frac{\hbar\kappa^2\omega_{\mathrm{p}}^2\sqrt{\Delta z C'^3}f_\Lambda}{32\pi\sqrt{2I_{\mathrm{c}}^3\varphi_0}}\cdot\frac{c_4}{c_2^{5/2}}\times$$
$$\times\left[|A_{\mathrm{p},0}|^2\hat{a}^\dagger_\omega\hat{a}_{\omega'}\mathrm{e}^{-\mathrm{i}[k(\omega)-k(\omega')]z+\mathrm{i}(\omega-\omega')t}+\mathrm{H.c.}\right]$$

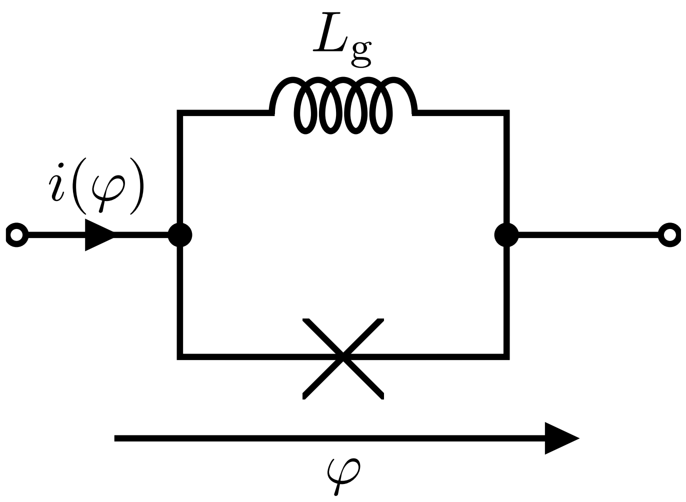
Three-wave mixing

Cross-phase modulation

- Dispersion factor from Yuan *et al.*, Phys. Rev. A **107** 022612 (2023)

$$f_\Lambda = \sqrt{\omega\omega'\Lambda(\omega)\Lambda(\omega')}\,[\Lambda(\omega)\Lambda(\omega')]^{\frac{1}{4}}$$

RF-SQUIDs



- Current-phase relation

$$i(\varphi) = \frac{I_\mathrm{c}}{\beta_\mathrm{L}}\varphi + I_\mathrm{c}[\sin(\varphi_{\mathrm{J},0} + \varphi) - \sin(\varphi_{\mathrm{J},0})]$$

- Induced flux

$$\varphi_{\mathrm{J},0} + \beta_\mathrm{L}\sin\varphi_{\mathrm{J},0} = \varphi_\mathrm{ext}$$

- Linear Inductance

$$i(\varphi) = I_\mathrm{c}\left[\frac{1}{\beta_\mathrm{L}} + \cos(\varphi_{\mathrm{J},0})\right](\varphi - \varphi^*) + \mathcal{O}(\varphi^2)$$

Nonlinear Coefficients

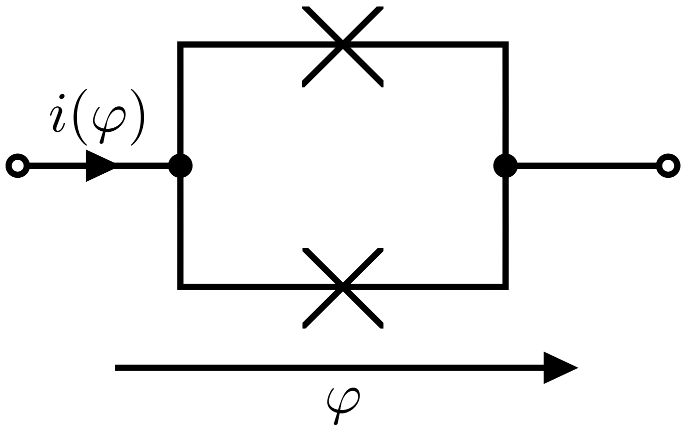
$$c_2 = \frac{1}{2}\left[\frac{1}{\beta_\mathrm{L}} + \cos(\varphi_{\mathrm{J},0}(\varphi_\mathrm{ext}))\right]\Bigg|_{\varphi_\mathrm{ext}=\varphi_\mathrm{B}},$$

$$c_3 = \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}\varphi_\mathrm{ext}}\cos(\varphi_{\mathrm{J},0}(\varphi_\mathrm{ext}))\Bigg|_{\varphi_\mathrm{ext}=\varphi_\mathrm{B}},$$

$$c_4 = \frac{1}{4}\frac{\mathrm{d}^2}{\mathrm{d}\varphi_\mathrm{ext}^2}\cos(\varphi_{\mathrm{J},0}(\varphi_\mathrm{ext}))\Bigg|_{\varphi_\mathrm{ext}=\varphi_\mathrm{B}},$$

Zorin, Phys. Rev. Appl. **6** 034006 (2016)

DC-SQUIDs



- Current-phase relation

$$i(\varphi) = 2I_\mathrm{c}\left|\cos\left(\frac{\varphi_\mathrm{ext}}{2}\right)\right|\sin(\varphi)$$

- Linear Inductance

$$i(\varphi) = 2I_\mathrm{c}\left|\cos\left(\frac{\varphi_\mathrm{ext}}{2}\right)\right|(\varphi - \varphi^*) + \mathcal{O}(\varphi^2)$$

Nonlinear Coefficients

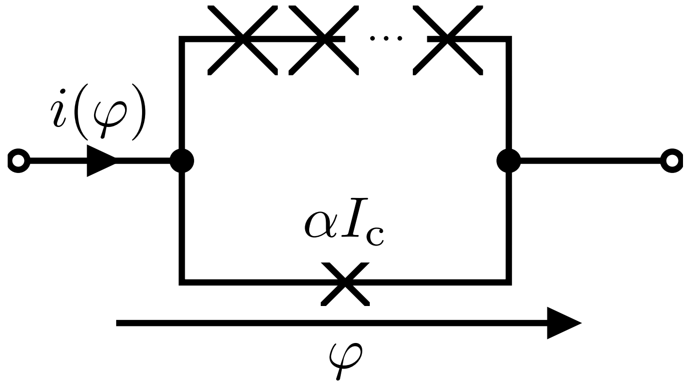
$$c_2 = \left|\cos\left(\frac{\varphi_\mathrm{ext}}{2}\right)\right|\Bigg|_{\varphi_\mathrm{ext}=\varphi_\mathrm{B}},$$

$$c_3 = \frac{\mathrm{d}}{\mathrm{d}\varphi_\mathrm{ext}}\left|\cos\left(\frac{\varphi_\mathrm{ext}}{2}\right)\right|\Bigg|_{\varphi_\mathrm{ext}=\varphi_\mathrm{B}},$$

$$c_4 = \frac{1}{2}\frac{\mathrm{d}^2}{\mathrm{d}{\varphi_\mathrm{ext}}^2}\left|\cos\left(\frac{\varphi_\mathrm{ext}}{2}\right)\right|\Bigg|_{\varphi_\mathrm{ext}=\varphi_\mathrm{B}}.$$

Zorin, Phys. Rev. Appl. **12** 044051 (2019)

SNAILs



- Current-phase relation

$$i(\varphi) = \alpha I_c \sin(\varphi) + I_c \sin\left(\frac{\varphi - \varphi_{ext}}{n}\right)$$

- Asymmetry ratio $\alpha = 0.29$
- Number of large junctions $n = 3$

- Linear Inductance

$$i(\varphi) \approx I_c \left[\alpha \cos(\varphi^*) + \frac{1}{n}\cos\left(\frac{\varphi^* - \varphi_{ext}}{n}\right)\right](\varphi - \varphi^*)$$

Nonlinear Coefficients

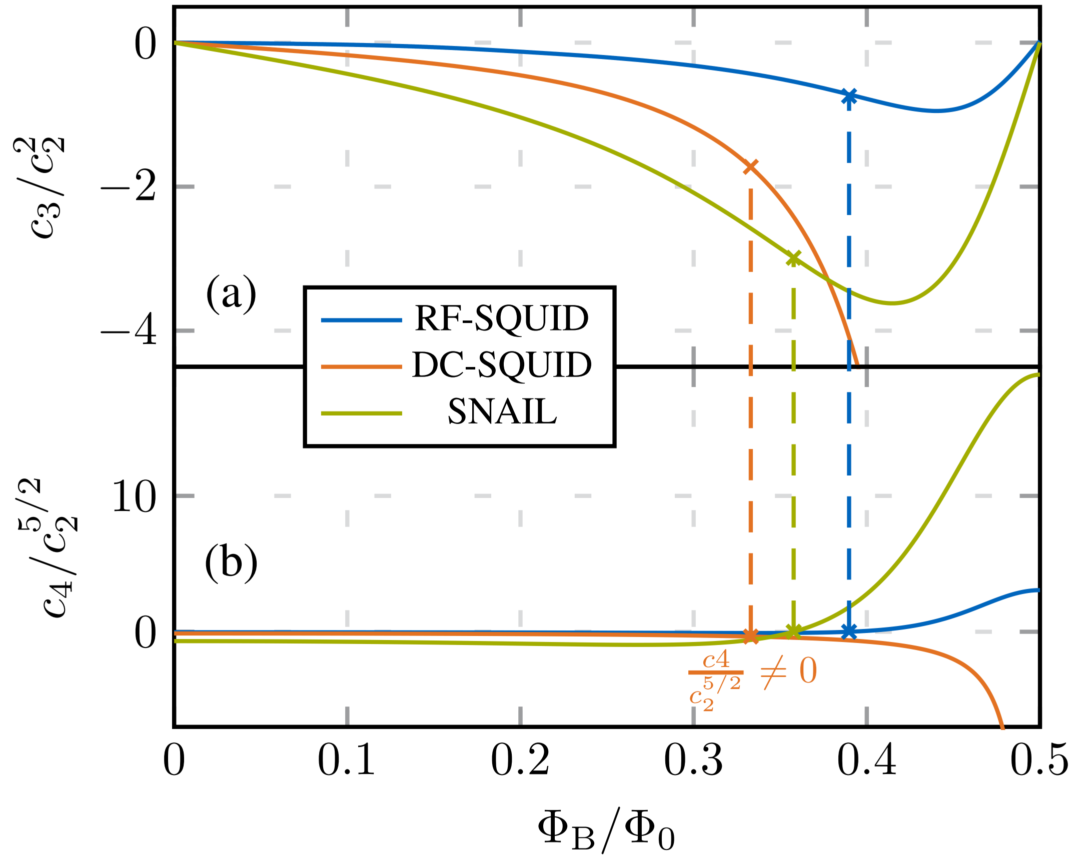
$$c_2 = \frac{1}{2}\left[\alpha \cos(\varphi^*) + \frac{1}{n}\cos\left(\frac{\varphi^* - \varphi_{ext}}{n}\right)\right]\Bigg|_{\varphi_{ext}=\varphi_B},$$

$$c_3 = \frac{1}{2}\frac{d}{d\varphi_{ext}}\left[\alpha \cos(\varphi^*) + \frac{1}{n}\cos\left(\frac{\varphi^* - \varphi_{ext}}{n}\right)\right]\Bigg|_{\varphi_{ext}=\varphi_B},$$

$$c_4 = \frac{1}{4}\frac{d^2}{d\varphi_{ext}^2}\left[\alpha \cos(\varphi^*) + \frac{1}{n}\cos\left(\frac{\varphi^* - \varphi_{ext}}{n}\right)\right]\Bigg|_{\varphi_{ext}=\varphi_B}.$$

Frattini *et al.*, Appl. Phys. Lett. **110**(22) 222603 (2017)

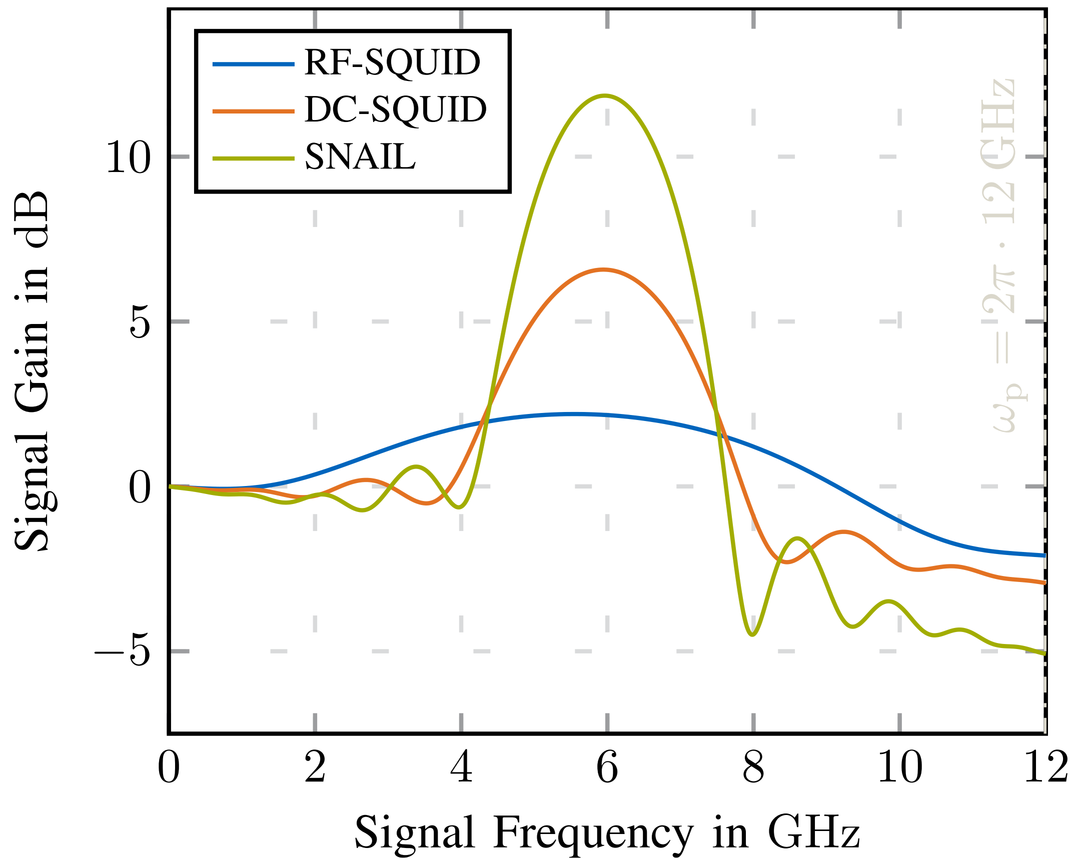
Results – Nonlinear Coefficients



- Largest 3WM coefficient for flux-driven SNAIL-based TWPAs
- No zero-XPM point for DC-SQUIDs
- Sign change at zero-XPM point for RF-SQUIDs and SNAILs which makes it possible to compensate for dispersion-induced phase mismatches

Ranadive *et al.*, Nat. Commun. **13** 1737 (2022)

Results – 3WM Gain



- Largest 3WM gain (>10 dB) for flux-driven SNAIL-based TWPA
- Losses have been considered by a substrate with $\tan\delta = 0.0025$
- **Advantage:** Better separation between signal and pump modes

Conclusion

- General method for quantum mechanical Hamiltonians and equations of motion for flux driven traveling-wave parametric amplifiers
- Different junction topologies have been studied, where Superconducting Nonlinear Asymmetric Inductive Elements turned out to be favorable…

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Munich Quantum Valley